



Problem of the Month

Problem 0: Toggling doors

September 2026

Hint

- (a) Solving this problem will come down to counting how many positive factors each door number has. It might be useful to determine the number of factors of the first few positive integers and see if you notice a pattern. Try determining the number of factors of the integers from 1 through 20.
 - (b) In this part, whether a door is open or closed only depends on how many odd positive factors it has.
 - (c) The following general fact may be useful in this or the other parts: Every integer n can be expressed in the form $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ where the p_i are distinct prime numbers and the e_i are positive integers. An integer d is a positive factor of n if and only if it can be expressed in the form $d = p_1^{f_1} p_2^{f_2} \cdots p_k^{f_k}$ where $0 \leq f_i \leq e_i$ for each i . This means that $\tau(n) = (e_1 + 1)(e_2 + 1) \cdots (e_k + 1)$. Can you see why?
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