The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

***cemc.uwaterloo.ca***

# *Galois Contest*

## *(Grade 10)*

***Wednesday, April 1, 2026***

***(in North America and South America)***

***Thursday, April 2, 2026***

***(outside of North America and South America)***

UNIVERSITY OF WATERLOO

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**Time:** 75 minutes 

*Do not open this booklet until instructed to do so.*

**Number of questions: 4** **Each question is worth 10 marks**

**Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.**

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

**WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.**

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked 💡, place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked ✍, provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the $x$-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. A *prime number* is a positive integer greater than 1 whose only positive divisors are 1 and itself. For example, 2 is a prime number since it is greater than 1 and its only positive divisors are 1 and 2.

   (a) What is the product of the three smallest prime numbers?

   (b) There are two integers $c$ with $1 \leq c \leq 20$ for which $\dfrac{c-3}{4}$ is a prime number. What are these two possible values of the integer $c$?

   (c) Determine all integers $d$ with $1 \leq d \leq 10$ for which $21d - 77$ is a prime number.

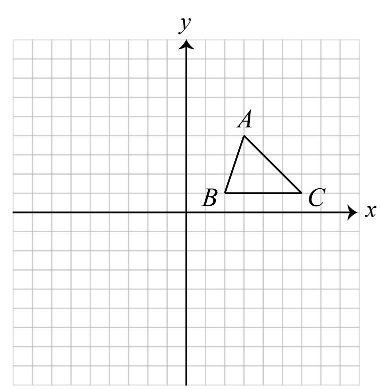
2. In the diagram, $\triangle ABC$ has vertices $A(3, 4)$, $B(2, 1)$ and $C(6, 1)$.

   (a) What is the area of $\triangle ABC$?

   (b) Point $D$ is the image of point $B$ after it is reflected in the $y$-axis. What is the area of $\triangle ADC$?

   (c) Point $E$ is the image of point $A$ after it is reflected in the horizontal line $y = -2$. What is the area of $\triangle EBC$?

   (d) Point $F$ is the image of point $A$ after it is reflected in the horizontal line $y = k$. Determine the two different values of $k$ for which the area of $\triangle FBC$ is equal to 12.



3. For every positive integer $a$, the units digits of $a^1$, $a^2$, $a^3$, $a^4$, $a^5$, $\ldots$, will form a repeating sequence. In each such sequence, the smallest number of consecutive units digits that repeat consecutively and indefinitely is at most 4. This number is called the *cycle length*. For example, when $a = 3$,

$$3^1 = 3,\ 3^2 = 9,\ 3^3 = 27,\ 3^4 = 81,\ 3^5 = 243,\ 3^6 = 729, \ldots$$

and the sequence of units digits is 3, 9, 7, 1, 3, 9, $\ldots$. In this example, the consecutive units digits that repeat are 3, 9, 7, 1, and so the cycle length is 4.

(a) What is the units digit of $3^{43}$?

(b) Determine the number of integers $j$ with $1 \leq j \leq 2026$ for which $4^j + 8^j$ is a multiple of 10.

(c) Determine the number of integers $k$ with $1 \leq k \leq 50$ for which $2^k + 3^k$ has the same units digit as $8^{2026k} + 9^{2026k}$.

4. (a) The set $R$ contains the points $(0, 0)$, $(0, 1)$, $(0, 4)$, $(0, 9)$, $(0, 16)$, and $(0, 25)$. The midpoint of every pair of distinct points from $R$ is plotted on the $xy$-plane. What is the number of distinct points that are plotted?

(b) The set $S$ contains exactly 20 distinct points, each of the form $(n, n + 2)$, where $n$ is an integer and $1 \leq n \leq 20$. The midpoint of every pair of distinct points from $S$ is plotted on the $xy$-plane. Determine the number of distinct points that are plotted.

(c) A set $T$ contains exactly 100 points having distinct $y$-coordinates. The midpoint of every pair of distinct points from $T$ is plotted on the $xy$-plane. Determine $m$, the minimum number of distinct points that could be plotted. A complete solution must

- include the value of $m$,
- describe a set of points $T$ for which the number of distinct midpoints is $m$, and
- provide an explanation of why, for all possible choices for the set $T$, fewer than $m$ distinct midpoints is not possible.

2026 Galois Contest (English)

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Thank you for writing the 2026 Galois Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2026.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Information about careers in and applications of mathematics and computer science

**For teachers...**

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