



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2026 Fryer Contest

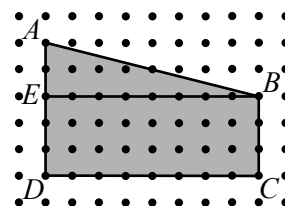
Wednesday, April 1, 2026
(in North America and South America)

Thursday, April 2, 2026
(outside of North America and South America)

Solutions

1. (a) The area of the rectangle without the square removed is $8 \times 6 = 48$.
 The area of the square that has been removed from the rectangle is $1 \times 1 = 1$.
 The area of the shaded shape in Figure 1 is $48 - 1 = 47$.
- (b) The removed triangle has base 6 and height 2, and thus has area $\frac{1}{2} \times 6 \times 2 = 6$.
 Following the work done in (a), the area of the shaded shape in Figure 2 is $48 - 6 = 42$.

(c) We begin by determining the area of $ABCD$. Construct line segment BE perpendicular to AD with E on AD , dividing $ABCD$ into right-angled triangle ABE and rectangle $EBCD$, as shown. The area of $\triangle ABE$ is $\frac{1}{2} \times 8 \times 2 = 8$. The area of rectangle $EBCD$ is $8 \times 3 = 24$, and so the area of $ABCD$ is $8 + 24 = 32$.



Therefore, the area of trapezoid $ABGH$ is $2 \times 32 = 64$.

Trapezoid $ABGH$ can be divided into right-angled triangle ABE and rectangle $EBGH$.

The area of $\triangle ABE$ is 8, and so the area of rectangle $EBGH$ is $64 - 8 = 56$.

The area of $EBGH$ is $EB \times BG = 8 \times BG = 56$, and so $BG = \frac{56}{8} = 7$.

2. (a) List P contains all positive integers from 1 to 2^{53} inclusive, and so P contains each integer of the form 2^k where k is a positive integer from 1 to 53 inclusive.

Thus, there are 53 such numbers in the list P .

- (b) Since $4^\ell = (2^2)^\ell = 2^{2\ell}$, then 4^ℓ is in list P exactly when $2^{2\ell} \leq 2^{53}$ or $2\ell \leq 53$ for positive integers ℓ . The largest positive integer ℓ for which $2\ell \leq 53$ is 26 (since $2 \times 26 = 52$, and $2 \times 27 = 54$).

Thus, for all positive integers $\ell \leq 26$, 4^ℓ is in list P , and so there are 26 such numbers.

We demonstrate this relationship between the powers of 2 and 4 in the table below.

2^k	2^1	2^2	2^3	2^4	2^5	2^6	...	2^{51}	2^{52}	2^{53}
$4^\ell = 2^{2\ell}$		$4^1 = 2^2$		$4^2 = 2^4$		$4^3 = 2^6$...		$4^{26} = 2^{52}$	

- (c) From (b), list P contains the 26 numbers $4^r = 2^{2r}$ for positive integers $r \leq 26$.

That is, each integer power of 4 in P is equal to a power of 2 whose exponent is an even positive integer.

Since $8^t = (2^3)^t = 2^{3t}$, then each integer power of 8 in P is equal to a power of 2 whose exponent is a positive integer multiple of 3.

Thus, any number in P that is an integer power of both 4 and 8 must be equal to a power of 2 whose positive integer exponent is both even and a multiple of 3, and therefore a multiple of 6.

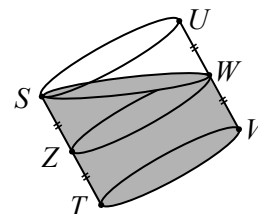
P contains all numbers 2^k for positive integers $k \leq 53$, and so P contains the following powers of 2 whose exponent is a multiple of 6: $2^6, 2^{12}, 2^{18}, 2^{24}, 2^{30}, 2^{36}, 2^{42}$, and 2^{48} .

For positive integers r and t , there are 8 numbers in P that can be written as both 4^r and as 8^t , and so there are $26 - 8 = 18$ numbers in P which can be written as 4^r but cannot be written as 8^t .

We demonstrate this relationship between the powers of 2, 4 and 8 in the table below.

$4^r = 2^{2r}$	4^1	4^2	$4^3 = 2^6$	4^4	4^5	$4^6 = 2^{12}$...	$4^{24} = 2^{48}$	4^{25}	4^{26}
$8^t = 2^{3t}$			$8^2 = 2^6$			$8^4 = 2^{12}$...	$8^{16} = 2^{48}$		
4^r not 8^t	4^1	4^2		4^4	4^5		...		4^{25}	4^{26}

3. (a) In the diagram, we let Z be the midpoint of ST . W is the midpoint of UV , and so SU , ZW and TV are parallel. A slice parallel to the base of the cylindrical container and passing through Z and W , divides the cylindrical container in half.

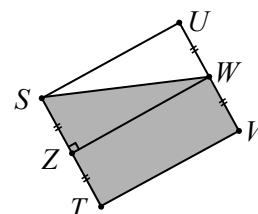


The bottom half of the cylindrical container is full of water, and so the volume of water in this portion is half the volume of the container.

The top half of the container is half-full of water, and so the volume of water in this portion is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ the volume of the container.

Thus, the fraction of the cylindrical container's volume that contains water is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

We can also visualize this by considering the 2-dimensional cross-section of the cylindrical container that passes through each of the six points shown.



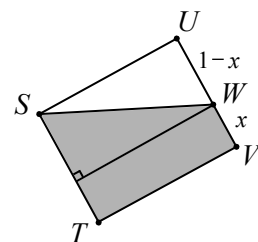
Can you see why using this 2-dimensional representation is equivalent to using the 3-dimensional representation to answer the question posed?

- (b) If $UV = 1$ and $WV = x$, then $WU = 1 - x$, where $0 \leq x < 1$.

In the diagram, the cylinder is sliced in a manner similar to what was done in part (a). The bottom portion of the container is full of water and the top portion is half-full of water. The fraction of the cylinder's volume that contains water is independent of the cylinder's radius. Thus, we choose the radius of the cylinder to be equal to 1.

So, the volume of the container is $\pi \times r^2 \times UV = \pi \times 1^2 \times 1 = \pi$.

The volume of water in the bottom portion of the container is $\pi \times r^2 \times WV = \pi \times 1^2 \times x = \pi x$. The volume of water in the top portion of the container is $\frac{1}{2} \times \pi \times r^2 \times WU = \frac{1}{2} \times \pi \times 1^2 \times (1 - x) = \frac{1}{2}\pi(1 - x)$.



Thus, the volume of water in the container is $\pi x + \frac{1}{2}\pi(1 - x)$.

Therefore, the fraction of the cylinder's volume that contains water is

$$\frac{\pi x + \frac{1}{2}\pi(1 - x)}{\pi} = x + \frac{1}{2}(1 - x) = x + \frac{1}{2} - \frac{1}{2}x = \frac{1}{2}x + \frac{1}{2},$$

where $0 \leq x < 1$.

- (c) The volume of a cylinder with radius 8 cm and height 12 cm is $\pi \times 8^2 \times 12 = 768\pi \text{ cm}^3$.

Thus, the fraction of this cylinder's volume that contains water is $\frac{624\pi \text{ cm}^3}{768\pi \text{ cm}^3} = \frac{13}{16}$.

Part (b) tells us that when $\frac{WV}{UV} = \frac{x}{1}$, or equivalently $WV = x \times UV$, the fraction of the cylinder's volume that contains water is $\frac{1}{2}x + \frac{1}{2}$.

Equating these two equivalent fractions and solving, we get

$$\begin{aligned}\frac{1}{2}x + \frac{1}{2} &= \frac{13}{16} \\ \frac{1}{2}x &= \frac{13}{16} - \frac{8}{16} \\ x &= \frac{5}{16} \times 2 \\ x &= \frac{5}{8}\end{aligned}$$

The height of the cylinder, UV , is 12 cm, and so $WV = \frac{5}{8} \times 12 \text{ cm} = \frac{15}{2} \text{ cm}$.

4. (a) Suppose the first five terms of the sequence are $x, y, 20, z, 57$.
 The sum of the third term, 20, and the fourth term, z , is equal to the fifth term, 57.
 Thus, $20 + z = 57$, or $z = 37$.
 Similarly, $y + 20 = 37$, and so the second term is $y = 37 - 20 = 17$.
 Finally, $x = 20 - y$, or $x = 20 - 17 = 3$.
 (The first five terms of this sequence are 3, 17, 20, 37, 57.)
- (b) The first six terms of this Dunbar sequence are $a, b, a + b, a + 2b, 2a + 3b, 3a + 5b$.
 Thus, we want to find all possible ordered pairs of positive integers (a, b) for which $a < b$ and $3a + 5b = 104$.
 Since $5b = 104 - 3a$, then $104 - 3a$ must be a multiple of 5 (since $5b$ is a multiple of 5).
 The smallest positive integer a for which $104 - 3a$ is a multiple of 5 is $a = 3$.
 When $a = 3$, $5b = 104 - 3(3) = 95$ and so $b = \frac{95}{5} = 19$.
 In this case, the first six terms of the sequence are 3, 19, 22, 41, 63, 104.
 The second smallest positive integer a for which $104 - 3a$ is a multiple of 5 is $a = 8$.
 When $a = 8$, $5b = 104 - 3(8) = 80$ and so $b = 16$.
 In this case, the first six terms of the sequence are 8, 16, 24, 40, 64, 104.
 The third smallest positive integer a for which $104 - 3a$ is a multiple of 5 is $a = 13$.
 When $a = 13$, $5b = 104 - 3(13) = 65$ and so $b = 13$.
 However, $(a, b) = (13, 13)$ does not satisfy the condition $a < b$, and for all values of a greater than 13, $a > b$.
 Thus, there are exactly two ordered pairs (a, b) satisfying the given conditions.
 These are $(3, 19)$ and $(8, 16)$.
- (c) *Solution 1*
 The first eight terms of this Dunbar sequence are $c, d, c + d, c + 2d, 2c + 3d, 3c + 5d, 5c + 8d, 8c + 13d$.
 Thus, we want to find all possible ordered pairs of positive integers (c, d) for which $c < d$ and $(5c + 8d)(8c + 13d) = 41\,440$.
 Suppose that we let the sixth, seventh and eighth terms of the sequence be the positive integers R, S and T , respectively.
 Then $R < S < T$ and $S \times T = 41\,440$.
 Since $S < T$ and $S \times T = 41\,440$, then $S^2 < 41\,440$ and so $S < \sqrt{41\,440}$ or $S \leq 203$ (since S is a positive integer).
 Also, since $R < S$ and $T = S + R$, then $T < 2S$.
 So, $41\,440 = S \times T < S \times (2S) = 2S^2$, and thus $41\,440 < 2S^2$ or $\sqrt{\frac{41\,440}{2}} < S$, which gives $144 \leq S$.

Summarizing, we have $S \times T = 41\,440$, where $144 \leq S \leq 203$.

Since S and T are positive integers, then (S, T) is a factor pair of $41\,440 = 2^5 \times 5 \times 7 \times 37$. If S is a factor of $2^5 \times 5 \times 7 \times 37$ and $144 \leq S \leq 203$, then the possible values of S are $2^2 \times 37 = 148$, $2^5 \times 5 = 160$, and $5 \times 37 = 185$. Thus, the factor pairs (S, T) are $(148, 280)$, $(160, 259)$, and $(185, 224)$.

If the seventh term is $S = 148$ and the eighth term is $T = 280$, then the sixth term is $280 - 148 = 132$, the fifth term is $148 - 132 = 16$, the fourth term is $132 - 16 = 116$, and the third term is $16 - 116 = -100$.

Each term must be a positive integer, and so $(S, T) \neq (148, 280)$.

For each of the two remaining possible factor pairs (S, T) , we similarly work backward in an attempt to determine (c, d) , the first two terms of the sequence.

We summarize this work in the table that follows.

Term number:	8	7	6	5	4	3	2	1
$(S, T) = (148, 280)$	280	148	132	16	116	-100		
$(S, T) = (160, 259)$	259	160	99	61	38	23	15	8
$(S, T) = (185, 224)$	224	185	39	147	-108			

Thus, $(c, d) = (8, 15)$ is the only pair for which the product of the seventh term and the eighth term is equal to $41\,440$.

Solution 2

We begin as we did in Solution 1 by letting the seventh term be $S = 5c + 8d$ and the eighth term be $T = 8c + 13d$, so $S \times T = 41\,440$.

Next, we work backward to determine the first term, c , and the second term, d , in terms of S and T .

With the eighth term equal to T and the seventh term equal to S , the sixth term is $T - S$. Then, the fifth term is $S - (T - S) = 2S - T$, the fourth term is $(T - S) - (2S - T) = 2T - 3S$, and the third term is $(2S - T) - (2T - 3S) = 5S - 3T$.

Finally, the second term is $(2T - 3S) - (5S - 3T) = 5T - 8S = d$, and the first term is $(5S - 3T) - (5T - 8S) = 13S - 8T = c$.

Given that $c > 0$, it follows that $13S - 8T > 0$ or $13S > 8T$.

Since $S \times T = 41\,440$, then $T = \frac{41\,440}{S}$. Substituting, we get $13S > 8 \times \frac{41\,440}{S}$.

S is a positive integer and so simplifying, we get $S^2 > \frac{8 \times 41\,440}{13}$ or $S \geq 160$.

Given that $c < d$, it follows that $13S - 8T < 5T - 8S$ or $21S < 13T$.

Substituting $T = \frac{41\,440}{S}$, we get $21S < 13 \times \frac{41\,440}{S}$.

S is a positive integer and so simplifying, we get $S^2 < \frac{13 \times 41\,440}{21}$ or $S \leq 160$.

Therefore, $S = 160$ and $T = \frac{41\,440}{160} = 259$.

Substituting the values of S and T , the first term is $c = 13S - 8T = 13(160) - 8(259) = 8$, and the second term is $d = 5T - 8S = 5(259) - 8(160) = 15$.

We can confirm that the first 8 terms of the sequence are 8, 15, 23, 38, 61, 99, 160, 259, and that $160 \times 259 = 41\,440$.