The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Wednesday, April 1, 2026
(in North America and South America)

Thursday, April 2, 2026
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked [lightbulb], place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked [pencil], provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. In the grids below, dots are 1 unit apart horizontally and 1 unit apart vertically. Shapes are created by connecting dots with straight lines.

   (a) In Figure 1, the shaded shape is a rectangle with width 8 and height 6 that has a 1×1 square removed. What is the area of the shaded shape in Figure 1?

   (b) In Figure 2, the shaded shape is a rectangle with width 8 and height 6 that has a triangle removed. What is the area of the shaded shape in Figure 2?

   (c) In Figure 3, trapezoid $ABCD$ has $AD = 5$, $BC = 3$ and $CD = 8$. Point G is placed vertically below C and point H is placed vertically below D, so that GH is parallel to CD and the area of trapezoid $ABGH$ is twice the area of $ABCD$. Determine the length of BG.

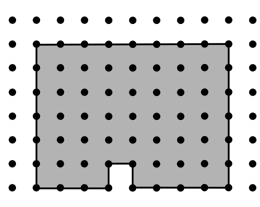

Figure 1

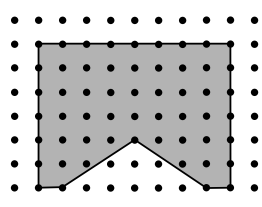

Figure 2

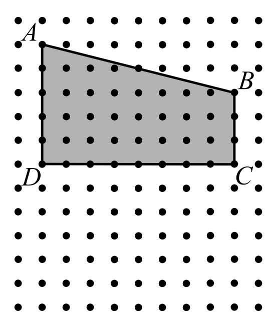

Figure 3

2. List P contains all positive integers from 1 to 2^{53}, inclusive.

   (a) How many numbers in list P can be written as 2^k where k is a positive integer?

   (b) Since $4 = 2^2$, every power of 4 can be written as a power of 2. For example, 4^3 can be written as $4^3 = \left(2^2\right)^3 = 2^{2\times 3} = 2^6$. In general, $4^n = \left(2^2\right)^n = 2^{2\times n} = 2^{2n}$. How many numbers in list P can be written as 4^ℓ where ℓ is a positive integer?

   (c) Determine how many numbers in list P can be written as 4^r where r is a positive integer, but cannot be written as 8^t where t is a positive integer.

3. An open-topped cylindrical container has points S and U placed diametrically opposite each other on the top edge. Points T and V on the bottom edge are vertically below S and U respectively, as shown in Figure 1. The cylinder is filled with water and then tipped so that some of the water spills out. The cylinder is then held in a *stationary tipped position* until the water stops spilling out and settles. At this time, the top surface of the water

- is horizontal,
- touches S, and
- touches UV at W, where $0 \le WV < UV$,

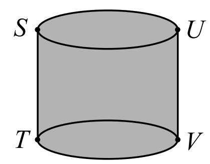

Figure 1

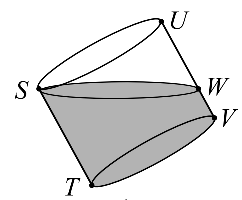

Figure 2

as shown in Figure 2. In the case when $WV = 0$, one half of the cylinder's volume contains water. In the questions that follow, each of the three cylinders has been filled with water, tipped, and is being held in a stationary tipped position.

(a) Suppose that $WV = WU$. What fraction of the cylinder's volume contains water?

(b) Suppose that $UV = 1$ and $WV = x$ where $0 \le x < 1$. Determine an expression for the fraction of the cylinder's volume that contains water, written in terms of x.

(c) Suppose that a cylinder with radius 8 cm and height 12 cm contains 624π cm^3 of water when held in a stationary tipped position. Determine the length of WV.

Note: A cylinder with radius r and height h has volume $\pi r^2 h$.

4. In a *Dunbar sequence*,

- each term is a positive integer,
- the second term is greater than the first term, and
- each term after the second is calculated by adding the two previous terms in the sequence.

For example, the first six terms of the Dunbar sequence with first term 2 and second term 5 are:

$$2, 5, 7, 12, 19, 31$$

(a) If the fifth term in a Dunbar sequence is 57 and the third term is 20, what is the first term in the sequence?

(b) Suppose that the first and second terms in a Dunbar sequence are a and b respectively. Determine all possible pairs (a, b) for which the sixth term of the sequence is equal to 104.

(c) Suppose that the first and second terms in another Dunbar sequence are c and d respectively. Determine all possible pairs (c, d) for which the product of the seventh term and the eighth term is equal to 41 440.

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Thank you for writing the 2026 Fryer Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2026.

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