The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

Euclid Contest

Tuesday, March 31, 2026
(in North America and South America)

Wednesday, April 1, 2026
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: $2\frac{1}{2}$ hours

Do not open this booklet until instructed to do so.

Number of questions: 10 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.

A Note about Bubbling

Please make sure that you have correctly coded your name, date of birth and grade on the Student Information Form, and that you have answered the question about residency.

1. (a) What is the integer t for which $\dfrac{2t}{3} + \dfrac{3t}{2} = 26$?

   (b) What is the integer x for which $\dfrac{3+x}{4} = \dfrac{6+x}{8}$?

   (c) Suppose that $y > 0$ and $\sqrt{3^2 + 4^2 + 12^2} = \sqrt{3^2 + 4^2} + \sqrt{y^2}$. Determine the value of y.

2. (a) The sum of the digits of the positive integer 2026 is $2 + 0 + 2 + 6 = 10$. What is the smallest integer $n > 2026$ whose digits also have a sum of 10?

   (b) The product of the digits of the integer 313 is $3 \cdot 1 \cdot 3 = 9$. How many integers between 100 and 999, including 313, have the property that the product of their digits is 9?

   (c) The sum of x, $3x$ and $4y$ is equal to 48. The average of x and y is equal to $3x$. Determine the ordered pair (x, y).

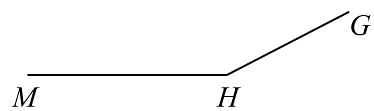
3. (a) What is the smallest positive integer n for which $\dfrac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{n}$ is equal to k^3 for some integer k?

   (b) What is the ordered pair (a, b) that satisfies both of the equations $3^{a+b} = 27$ and $a - b = -5$?

   (c) For some real number c, the parabola with equation $y = -x^2 + 7x + c$ intersects the x-axis at points $P(10, 0)$ and Q. If the parabola intersects the y-axis at R, determine the area of $\triangle PQR$.

4. (a) In January, Rebecca measured the temperature in Yellowknife every day at 11:00 a.m. The average of these 31 temperatures was $-20°$C. The average of the temperatures from the first 21 days was $-15°$C. What was the average of the temperatures from the last 10 days?

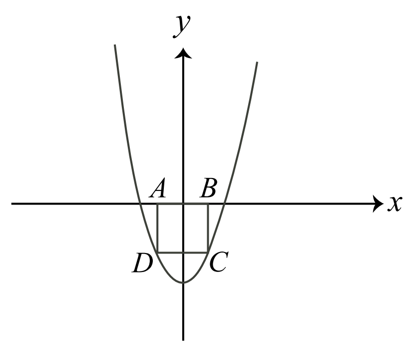
(b) McKayla runs to her grandmother's house and then runs home along the same straight road. The route from McKayla's house, M, to her grandmother's house, G, is on flat ground from M to H, and then uphill from H to G, as shown in the cross-section below. The distance from M to H to G is 10 km. (That is, $MH + HG = 10$ km.)



McKayla runs on flat ground at 12 km/h, uphill at 10 km/h, and downhill at 15 km/h. It takes 54 minutes for her to run from M to H to G. Determine the number of minutes that it takes for her to run from G to H to M.

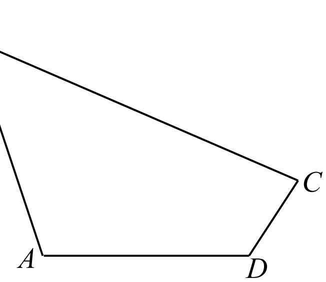
5. (a) Two fair dice, called D_1 and D_2, each have six faces. D_1 has the numbers 1, 2, 3, 4, 5, 6 on its faces. D_2 has a 1 on some of its faces and a 2 on its remaining faces. When D_1 and D_2 are rolled, the probability that the sum of the numbers on the top faces is a prime number is $\frac{23}{36}$. How many faces on D_2 have the number 1 on them?

(b) In the diagram, square $ABCD$ has A and B on the x-axis and C and D below the x-axis on the parabola with equation $y = x^2 - 4$. Determine the area of $ABCD$, writing your answer in the form $r - \sqrt{t}$ for some positive integers r and t.



6. (a) Xander, Yasmin and Zhe each have a rope. Xander's rope is 10 m long. Yasmin's rope is $n\%$ longer than Xander's rope. Zhe's rope is $(2n)\%$ longer than Yasmin's rope. Zhe's rope is $(3.14n)\%$ longer than Xander's rope. If $n > 0$, what is the value of n?

(b) In the diagram, quadrilateral $ABCD$ has $AB = AD = 4$. Also, $\angle ABC = 45°$ and $\angle CDA = 135°$. Determine the exact value of $BC - CD$.



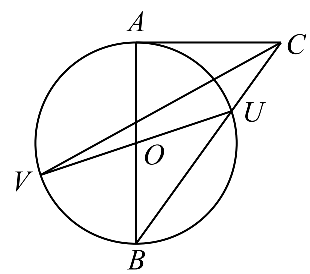
7. (a) In a garden, there are roses and carnations, and no other kind of flower. Each flower is either yellow or white. Half of the yellow flowers are roses, $\frac{1}{4}$ of the roses are yellow, and $\frac{1}{4}$ of all of the flowers are white carnations. What fraction of all of the flowers are yellow roses?

(b) The function f is defined for every real number $x > 0$ in the following way:

$$f(x) = \begin{cases} 0 & \text{if } 0 < x \leq 1 \\ 1 + f(\log_2 x) & \text{if } x > 1 \end{cases}$$

Determine the number of positive integers n for which $f(n) = 4$.

8. (a) In the diagram, the circle has centre O and radius 2. Points A, B, U, and V are on the circle so that AB and UV are diameters. Point C is outside the circle so that AC is tangent to the circle at A and so that BC intersects the circle at U. If $BU = 2UC$, determine the area of $\triangle CUV$.



(b) Determine all positive integers k for which there are exactly 100 non-congruent triangles that have positive integer side lengths, one side length equal to k, perimeter equal to $3k$, and one angle with measure greater than $90°$.

9. (a) Suppose that $\triangle XYZ$ has $XY = XZ$ and $YZ = b$. If the area of $\triangle XYZ$ is 40 and the perimeter of $\triangle XYZ$ is 32, determine a cubic polynomial f with integer coefficients and with the property that $f(b) = 0$.

(b) Consider positive real numbers A and P. If there exists an isosceles triangle with area A and perimeter P, prove that there exist at most two non-congruent isosceles triangles with area A and perimeter P.

(c) Determine positive integers A and P for which there exist two non-congruent isosceles triangles XYZ with $XY = XZ$, integer side lengths less than 300, area A, and perimeter P, and with the property that the length of YZ in exactly one of the two triangles is a multiple of 7.

10. Suppose that n is a positive integer with $n \geq 5$. An arrangement $a_1, a_2, \ldots, a_{n-1}, a_n$ of the n positive integers $1, 2, \ldots, n-1, n$ is said to have an *internal peak* at position t (with $2 \leq t \leq n-1$) if $a_{t-1} < a_t$ and $a_t > a_{t+1}$. For example, the arrangement $2, 3, 7, 1, 5, 4, 6, 8$ of the integers $1, 2, 3, 4, 5, 6, 7, 8$ has an internal peak at position 3, an internal peak at position 5, and no other internal peaks.

(a) Determine the number of arrangements of $1, 2, 3, 4, 5$ that have an internal peak at position 3 and no other internal peaks.

(b) Suppose that $n \geq 5$. Determine, in terms of n, an expression in closed form for the number of arrangements of $1, 2, \ldots, n-1, n$ that have an internal peak at position 2 and no other internal peaks.

(c) Suppose that $n \geq 5$. Determine, in terms of n, an expression in closed form for the number of arrangements of $1, 2, \ldots, n-1, n$ that have an internal peak at position 3 and no other internal peaks.

Note 1: Depending on your approach, the following formula might be useful.

For every positive integer r,

$$\sum_{k=0}^{r} k2^k = 0 \cdot 2^0 + 1 \cdot 2^1 + 2 \cdot 2^2 + \cdots + r2^r = (r-1)2^{r+1} + 2$$

Note 2: If you are unsure what is meant by "closed form" in parts (b) and (c), consider the three equal expressions in **Note 1** as an example:

- The expression $\sum_{k=0}^{r} k2^k$ *is not* a closed form.
- The expression $0 \cdot 2^0 + 1 \cdot 2^1 + 2 \cdot 2^2 + \cdots + r2^r$ *is not* a closed form.
- The expression $(r-1)2^{r+1} + 2$ *is* a closed form.

2026 Euclid Contest (English)

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For students...

Thank you for writing the 2026 Euclid Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

If you are graduating from secondary school, good luck in your future endeavours! If you will be returning to secondary school next year, encourage your teacher to register you for the 2026 Canadian Senior Mathematics Contest, which will be written in November 2026.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Information about careers in and applications of mathematics and computer science

For teachers...

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- Look at our free online courseware
- Use our free Problem Set Generator to create problem sets for curriculum support and enrichment
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results