The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2026 Cayley Contest

(Grade 10)

Wednesday, February 25, 2026
(in North America and South America)

Thursday, February 26, 2026
(outside of North America and South America)

Solutions

1. Evaluating, we get $13 + 15 + 17 = 28 + 17 = 45$.

   Answer: (B)

2. Written numerically, one million is 1 000 000 which is equal to 10^6.

   Answer: (D)

3. *Solution 1*
   Since $4x + 1 = 13$, then $3 \times (4x + 1) = 3 \times 13$, and so $12x + 3 = 39$.

   Solution 2
   Solving $4x+1 = 13$, we get $4x = 12$ and so $x = 3$. Thus, the value of $12x+3 = 12(3)+3 = 39$.

   Answer: (E)

4. Since the number line between 0 and 6 is divided into 12 equal pieces, then each piece has length $\frac{6}{12} = 0.5$.
   The location of M is 2 such pieces right of 0, and so the value of M is $2 \times 0.5 = 1.0$.
   The location of N is 3 such pieces left of 6, and so the value of N is $6 - (3 \times 0.5) = 4.5$.
   The value of $N - M = 4.5 - 1.0 = 3.5$.

   Answer: (B)

5. Running every second day, Carley ran on May 18, 20, 22, and 24. Between May 18th and 25th inclusive, Carley ran on 4 days.

   Answer: (E)

6. Of Jin's 40 cupcakes, 75% or $\frac{3}{4}$ are chocolate, and so Jin has $40 \times \frac{3}{4} = 30$ chocolate cupcakes and $40 - 30 = 10$ vanilla cupcakes.
   Since Jin has $30 - 10 = 20$ more chocolate cupcakes than she has vanilla cupcakes, then Jin must bake 20 more vanilla cupcakes so that half of the cupcakes are vanilla.

   Answer: (B)

7. Since $9 = 3 \times 3$, then 9 leaves remainder 0 when divided by 3. Since $9 = 2 \times 4 + 1$, then 9 leaves remainder 1 when divided by 4. The remainders upon division by 3 and also division by 4 for each of the other given numbers are in the table that follows.

	9	11	10	13	8
remainder when divided by 3	0	2	1	1	2
remainder when divided by 4	1	3	2	1	0

   Of the given integers, 13 has the same remainder when it is divided by 3 as when it is divided by 4.

   Answer: (D)

8. When a point (x, y) is shifted left 5 units, the coordinates of the resulting point are $(x - 5, y)$.
   Thus when $Q(4, 2)$ is shifted left 5 units, the coordinates of the resulting point are $(-1, 2)$.
   When a point (x, y) is shifted down 6 units, the coordinates of the resulting point are $(x, y-6)$.
   Thus when $(-1, 2)$ is shifted down 6 units, the coordinates of the resulting point are $R(-1, -4)$.

   Answer: (A)

9. *Solution 1*
If the length, width and height of the rectangular prism are the integers ℓ cm, w cm and h cm respectively, then the areas of three of its faces are ℓw cm^2, ℓh cm^2 and wh cm^2.
Thus, each pair of areas shares a common factor, and the common factors are the dimensions of the prism, ℓ, w and h.
The numbers 12 and 15 have 1 and 3 as common factors.
If one of ℓ, w or h is equal to 1, then the other two dimensions are 12 and 15, but this is not possible since 12×15 is not equal to 20.
If one of the dimensions is equal to 3, then the other dimensions are $\frac{12}{3} = 4$ and $\frac{15}{3} = 5$, whose product gives the required 20.
Thus, ℓ, w and h are equal to 3, 4 and 5, in some order, and so the volume of the rectangular prism is $V = 3 \text{ cm} \times 4 \text{ cm} \times 5 \text{ cm}$ or 60 cm^3.

Solution 2
If the length, width and height of the rectangular prism are the integers ℓ cm, w cm and h cm respectively, then the areas of three of its faces are ℓw cm^2, ℓh cm^2 and wh cm^2.
Thus ℓw, ℓh and wh are equal to 12, 20 and 15 in some order, and so $(\ell w)(\ell h)(wh) = 12 \times 20 \times 15$ or $\ell^2 w^2 h^2 = 3600$. Therefore, $(\ell wh)^2 = 3600$, which gives $\ell wh = 60$.
Measured in cm^3, the volume of the prism is $V = \ell wh = 60$.

Answer: (A)

10. Initially, the mean of the three visible numbers is $\frac{3+7+8}{3} = \frac{18}{3} = 6$. After the 3 is flipped, the new mean is 7.
If the number on the other side of the card with the 3 is x, then $\frac{x+7+8}{3} = 7$ or $x + 15 = 3 \times 7$ and so $x = 21 - 15 = 6$. Note that the mean of the numbers on the three cards increased by 1 when the 3 was flipped, and so it follows that the sum of the three visible numbers increased by 3. So, the new card (the opposite side of the card with the 3) must be $3 + 3 = 6$.
Similarly, since the mean increased by 1 again when the 7 was flipped, the number on the opposite side of the card with the 7 is $7 + 3 = 10$.
Finally, the mean increased by 1 when the 8 was flipped and so the number on the opposite side of the card with the 8 is $8 + 3 = 11$.

Answer: (A)

11. *Solution 1*
In $\triangle ABC$, $\angle ACB = 180^\circ - \angle ABC - \angle BAC = 180^\circ - 90^\circ - 42^\circ = 48^\circ$.
Since $\angle BCD = 90^\circ$, then $\angle ACD = \angle BCD - \angle ACB = 90^\circ - 48^\circ = 42^\circ$.
Since $\angle ACE = 90^\circ$, then $\angle DCE = \angle ACE - \angle ACD = 90^\circ - 42^\circ = 48^\circ$.

Solution 2
Since $\angle ABC = \angle BCD = 90^\circ$, then AB is parallel to DC.
Thus, $\angle ACD = \angle BAC = 42^\circ$ by a parallel lines theorem (Z pattern, alternate interior angles).
Since $\angle ACE = 90^\circ$, then $\angle DCE = \angle ACE - \angle ACD = 90^\circ - 42^\circ = 48^\circ$.

Solution 3
Since $\angle BAD = 90^\circ$, then $\angle CAD = \angle BAD - \angle BAC = 90^\circ - 42^\circ = 48^\circ$.
In $\triangle ADC$, $\angle ACD = 180^\circ - \angle ADC - \angle CAD = 180^\circ - 90^\circ - 48^\circ = 42^\circ$.
Since $\angle ACE = 90^\circ$, then $\angle DCE = \angle ACE - \angle ACD = 90^\circ - 42^\circ = 48^\circ$.

Answer: (C)

12. In March, Max read $\frac{1}{3}$ of the b books, meaning that he had $1-\dfrac{1}{3}=\dfrac{2}{3}$ of the b books, or $\dfrac{2}{3}b$ books, left to read. In April, he read 5 more books, thus leaving $\dfrac{2}{3}b-5$ books left to read. Max then had 7 books left to read and so $\dfrac{2}{3}b-5=7$. Solving, we get $\dfrac{2}{3}b=12$ or $2b=12\times 3$ and so $b=\dfrac{36}{2}=18$.

Answer: (A)

13. The area of a circle with radius K is 15π, and so $\pi\times K^2=15\pi$ or $K^2=15$.
The area of a circle with radius $2K$ is $\pi\times(2K)^2=\pi\times 4K^2$.
Substituting $K^2=15$, we get the area of a circle with radius $2K$ is $\pi\times 4(15)=60\pi$.

Answer: (E)

14. *Solution 1*
If $\dfrac{2m-n}{n}=\dfrac{3}{5}$, then $5(2m-n)=3n$ or $10m-5n=3n$. Simplifying, we get $10m=8n$ and so $\dfrac{m}{n}=\dfrac{8}{10}=\dfrac{4}{5}$.

Solution 2
If $\dfrac{2m-n}{n}=\dfrac{3}{5}$, then $\dfrac{2m}{n}-\dfrac{n}{n}=\dfrac{3}{5}$ or $\dfrac{2m}{n}-1=\dfrac{3}{5}$. Simplifying, we get $\dfrac{2m}{n}=\dfrac{8}{5}$, and so $\dfrac{m}{n}=\dfrac{8}{5}\times\dfrac{1}{2}=\dfrac{4}{5}$.

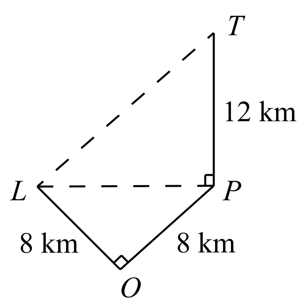
Answer: (A)

15. Suppose the diameter of each semi-circle is x.
That is, $PQ=RS=TU=x$, and so $PU=PQ+QT+TU=x+5+x=2x+5$.
Also, $PU=PR+RS+SU=8+x+8=x+16$.
Thus, $2x+5=x+16$ or $x=11$, and so $PQ=11$.

Answer: (D)

16. Let O be the point from which the two boats begin, let L be the point that The Luna reaches after the first hour, let P be the point that The Tuna reaches after the first hour, and let T be the point that The Tuna reaches after it travels north for 90 minutes from P.

The Luna travels from O to L in 1 hour at a speed of 8 km/h, so the length of OL is 8 km.
The Tuna travels from O to P in 1 hour at a speed of 8 km/h, so the length of OP is 8 km.
The Tuna travels from P to T in 90 minutes or 1.5 hours at a speed of 8 km/h, so the length of PT is 8 km/h $\times$ 1.5 h $=$ 12 km.



The direction from O to L is northwest, and the direction from O to P is northeast. Each of these directions are $45°$ from north, and so we conclude that $\angle LOP = 45° + 45° = 90°$.

In the first hour, The Luna and The Tuna each traveled the same distance at an angle of $45°$ from north, and so the two boats were at exactly the same latitude when they reached points L and P, respectively. Put differently, The Tuna was directly east of The Luna after the first hour. Since The Tuna then travels north, we conclude that $\angle LPT = 90°$.

We have that $\triangle LOP$ is right-angled at O and $\triangle LPT$ is right-angled at P.
By the Pythagorean theorem, we get $OL^2 + OP^2 = LP^2$ and $LP^2 + PT^2 = LT^2$.
Substituting the expression for LP^2 from the first of these equations into the second, and using the lengths calculated above, we have

$$\begin{aligned} LT^2 &= OL^2 + OP^2 + PT^2 \\ &= 8^2 + 8^2 + 12^2 \\ &= 64 + 64 + 144 \\ &= 272 \end{aligned}$$

Since $LT > 0$, we have $LT = \sqrt{272}$. To the nearest kilometre, the distance between the two boats is 16.

Answer: (A)

17. If the positive two-digit integer n has tens digit x and units digit y, then $n = 10x + y$ and $s = x + y$. Then $n - s = (10x + y) - (x + y) = 9x$, and so $n - s$ is a multiple of 9 (since x is a positive integer). Of the answers given, the only multiple of 9 is 54.
(If for example $n = 62$, then $n - s = 62 - (6 + 2) = 54$.)

Answer: (D)

18. The grasshopper begins at the origin, $(0, 0)$.
At $(0, 0)$, the sum of the x and y-coordinates is $x + y = 0 + 0 = 0$.
Each jump (up, down, left or right) increases or decreases either the x-coordinate or the y-coordinate by 1, and thus increases or decreases $x + y$ by 1.
Thus each jump changes the parity of $x + y$ (from even to odd or from odd to even).
Since $x + y$ is even at $(0, 0)$, then after the grasshopper jumps exactly 6 times, $x + y$ is even at its final location (x, y).
We begin by determining at which points (x, y) the grasshopper can finish, where $x \geq 0$ and $y \geq 0$ (in the first quadrant or on the axes bordering the first quadrant).
Since the grasshopper jumps 6 times, then $x + y$ is at most 6.
For $x + y = 6$, the grasshopper can, for example, jump 6 units right to end at $(6, 0)$, or jump 5 units right and up 1 to end at $(5, 1)$. Continuing in this way with $x + y = 6$, the remaining possible ending points are $(4, 2)$, $(3, 3)$, $(2, 4)$, $(1, 5)$, and $(0, 6)$.
At the grasshopper's ending point (x, y), can $x + y = 4$? (Recall that $x + y$ is even so we may ignore $x + y = 5$, for example.) The grasshopper could, for example, jump 1 unit right, then 1 unit left, returning to $(0, 0)$ with 4 jumps remaining. Thus for $x + y = 4$, the possible ending points are $(4, 0)$, $(3, 1)$, $(2, 2)$, $(1, 3)$, and $(0, 4)$.
When $x + y = 2$, the possible ending points are $(2, 0)$, $(1, 1)$, and $(0, 2)$.
And finally, the grasshopper could jump right 3 times and left 3 times to end at $(0, 0)$.
To summarize the case for which $x \geq 0$ and $y \geq 0$, the grasshopper can end at

(i) 9 points with $x > 0$ and $y > 0$: $(5, 1)$, $(4, 2)$, $(3, 3)$, $(2, 4)$, $(1, 5)$, $(3, 1)$, $(2, 2)$, $(1, 3)$, $(1, 1)$,

(ii) 6 points on the positive x-axis or positive y-axis: $(2, 0)$, $(4, 0)$, $(6, 0)$, $(0, 2)$, $(0, 4)$, $(0, 6)$,

(iii) the point $(0, 0)$.

By symmetry, the grasshopper can end at the 9 points that correspond to (i) in each of the other 3 quadrants for a total of $9 \times 4 = 36$ points.
By symmetry, the grasshopper can end at the 6 points that correspond to (ii) on the negative x-axis or the negative y-axis for a total of $6 \times 2 = 12$ points.
Thus, $N = 36 + 12 + 1 = 49$.

Answer: (D)

19. Since the mean of the first three rolls is 3, then the sum of the first three rolls is $3 \times 3 = 9$.
The results of the 4th, 5th and 6th rolls are a, b and c, and so the mean of all six rolls is $\frac{9 + a + b + c}{6}$. For this mean to be an integer, $9 + a + b + c$ must be a multiple of 6.
Each roll is a positive integer between 1 and 6 inclusive, and so $a+b+c$ is at least $1+1+1 = 3$ and at most $6 + 6 + 6 = 18$.
Therefore, $9 + a + b + c$ is at least $9 + 3 = 12$ and at most $9 + 18 = 27$.
The multiples of 6 between 12 and 27 inclusive are 12, 18 and 24, and so the mean is an integer exactly when $a + b + c = 3$ or $a + b + c = 9$ or $a + b + c = 15$.
Next, we count the number of ordered triples (a, b, c) for each of these 3 cases.

Case 1: $a + b + c = 3$
There is exactly 1 ordered triple, $(a, b, c) = (1, 1, 1)$, in this case.

Case 2: $a + b + c = 9$
If $a = 6$, then $b + c = 3$ and so $(b, c) = (1, 2)$ or $(b, c) = (2, 1)$.
If $a = 5$, then $b + c = 4$ and so $(b, c) = (1, 3)$ or $(b, c) = (2, 2)$ or $(b, c) = (3, 1)$.
We continue in this way and summarize the results in the table that follows.

Value of a	Value of $b + c$	Possible ordered pairs (a, b)	Number of ordered triples
6	3	$(1, 2)$, $(2, 1)$	2
5	4	$(1, 3)$, $(2, 2)$, $(3, 1)$	3
4	5	$(1, 4)$, $(2, 3)$, $(3, 2)$, $(4, 1)$	4
3	6	$(1, 5)$, $(2, 4)$, $(3, 3)$, $(4, 2)$, $(5, 1)$	5
2	7	$(1, 6)$, $(2, 5)$, $(3, 4)$, $(4, 3)$, $(5, 2)$, $(6, 1)$	6
1	8	$(2, 6)$, $(3, 5)$, $(4, 4)$, $(5, 3)$, $(6, 2)$	5

In total, there are $2 + 3 + 4 + 5 + 6 + 5 = 25$ such ordered triples in this case.

Case 3: $a + b + c = 15$
We count in a manner similar to that in Case 2, and summarize the results in the table that follows.

Value of a	Value of $b + c$	Possible ordered pairs (a, b)	Number of ordered triples
6	9	$(3, 6)$, $(4, 5)$, $(5, 4)$, $(6, 3)$	4
5	10	$(4, 6)$, $(5, 5)$, $(6, 4)$	3
4	11	$(5, 6)$, $(6, 5)$	2
3	12	$(6, 6)$	1

If $a < 3$, then $b + c > 12$ which is not possible.
In total, there are $4 + 3 + 2 + 1 = 10$ such ordered triples in this case.

The number of ordered triples (a, b, c) for which the mean of all six rolls is an integer is $1 + 25 + 10 = 36$.

Answer: (E)

20. The choice of 5 letters for each of the 5 positions within the string gives a total of $5^5 = 3125$ different strings in the list.
Of these strings, $\frac{1}{5}$ begin with the letter A, $\frac{1}{5}$ begin with the letter B, and so on to E.
That is, there are $\frac{1}{5} \times 5^5 = 5^4 = 625$ strings that begin with each of the letters A through E.
Thus, the $3 \times 625 = 1875$th string is the last string that begins with C (this is $CEEEE$), and the $4 \times 625 = 2500$th string is the last string that begins with D (this is $DEEEE$).
This tells us that the first letter in the 2026th string is D.

Of the 625 strings that begin with D, $\frac{1}{5}$ have second letter A, $\frac{1}{5}$ have second letter B, and so on to E. That is, there are $\frac{1}{5} \times 625 = 125$ strings having each of the second letters A through E.
Thus, the $1875 + 125 = 2000$th string is the last string that begins with D and has second letter A (this is $DAEEE$), and the $1875 + 2 \times 125 = 2125$th string is the last string that begins with D and has second letter B (this is $DBEEE$).
This tells us that the 2001st string is $DBAAA$, and the first two letters in the 2026th string are DB.

Of the 125 strings that begin with DB, $\frac{1}{5}$ have third letter A, $\frac{1}{5}$ have third letter B, and so on to E. That is, there are $\frac{1}{5} \times 125 = 25$ strings beginning with DBA. The 2001st of these is $DBAAA$ and the $2000 + 25 = 2025$th is $DBAEE$. Thus, the 2026th string is the next string following $DBAEE$ which is $DBBAA$, and so the 2026th string in the list has 2 Bs.

Answer: (C)

21. The shaded region is a 16×16 square with four quarter circles, each with radius 8, removed. Thus, the region has the same area as that of a 16×16 square with a circle of radius 8 removed.

A 16×16 square has area 256. A circle with radius 8 has area 64π. Thus, the area of the shaded region is

$$256 - 64\pi \approx 54.94$$

Rounded to the nearest integer, the area is 55.

Answer: 55

22. Let y be the number of people who bought a drink. It is given that $y > 40$.
The number of people who bought popcorn but did not buy a drink is $47 - x$.
The number of people who bought a drink but did not buy popcorn is $y - x$.
The number of people who bought both is given to be x, and the number of people who bought neither is given to be $2x$.
Every person falls into exactly one of the cases above, so we have that

$$100 = (47 - x) + (y - x) + x + 2x$$

which can be solved for y to get $y = 53 - x$. It is given that $y > 40$, so we must have $53 - x > 40$, which can be rearranged to get $x < 53 - 40$ or $x < 13$. Since x is an integer, we must have $x \leq 12$.

Note that if $x = 12$, the purchases satisfy the conditions:

purchases	number of people
popcorn and drink	12
popcorn and no drink	35
drink and no popcorn	29
nothing	24

ANSWER: 12

23. The integers $2r - 21$, $3r - 1$, $r + 12$, and $3r - 17$ are equal to the integers p, $2p$, q, and $2q$ (in some order), and so we get the equivalent equations

$$\begin{aligned}(2r - 21) + (3r - 1) + (r + 12) + (3r - 17) &= p + 2p + q + 2q \\ 9r - 27 &= 3p + 3q \\ 3r - 9 &= p + q \qquad (*)\end{aligned}$$

It is given that p and q are two of $2r - 21$, $3r - 1$, $r + 12$, $3r - 17$. There are 6 possibilities for the two integers that are equal to p and q.

- If p and q are $2r - 21$ and $3r - 1$ (in some order), $p + q = (2r - 21) + (3r - 1) = 5r - 22$. We also know that $p + q = 3r - 9$, so $5r - 22 = 3r - 9$, which can be solved for r to get $r = \frac{13}{2}$, which is not an integer. We conclude that p and q cannot be $2r - 21$ and $3r - 1$.
- If p and q are $2r - 21$ and $r + 12$, then $p + q = (2r - 21) + (r + 12) = 3r - 9$, which agrees with equation $(*)$. (This will end up being the only possibility.)
- If p and q are $2r - 21$ and $3r - 17$, then $p + q = 5r - 38$. Again using that $p + q = 3r - 9$, we get $3r - 9 = 5r - 38$. Solving for r gives $r = \frac{29}{2}$, which is not an integer.
- If p and q are $3r - 1$ and $r + 12$, then one can check that $r = -20$, which is not a positive integer.
- If p and q are $3r - 1$ and $3r - 17$, then $r = 3$. If $r = 3$, then $2r - 21$ is negative, but this is not allowed since it is one of p, q, $2p$, and $2q$, which all must be positive.
- If p and q are $r + 12$ and $3r - 17$, then $r = -4$, which is not a positive integer.

We conclude that p and q must be $2r - 21$ and $r + 12$ in some order, which means $2p$ and $2q$ must be $3r - 1$ and $3r - 17$ in some order. Noting that p and q are interchangeable here, we can assume that $p = 2r - 21$ and $q = r + 12$, and thus there are 2 cases to consider.

Case 1: $p = 2r - 21$ and $2p = 3r - 1$

If $p = 2r - 21$, then $2p = 2(2r - 21) = 4r - 42$. Equating the two expressions for $2p$ and solving, we get $4r - 42 = 3r - 1$, and so $r = 41$.

Case 2: $p = 2r - 21$ and $2p = 3r - 17$

Equating the two expressions for $2p$ and solving, we get $4r - 42 = 3r - 17$, and so $r = 25$.

The sum of all possible values of r is $41 + 25 = 66$.

ANSWER: 66

24. Since they are alternate angles, we have that $\angle FEG = \angle ACG$ and $\angle EFG = \angle CAG$. Each of AC and FE are half the edge length of the same cube, so $AC = FE$. By angle-side-angle congruence, $\triangle EFG$ is congruent to $\triangle CAG$.

From this congruence, we get that $CG = EG$, which means G is the midpoint of CE. Similarly, H is the midpoint of CD.

Label the midpoint of CF by J. Then we have $\frac{CG}{CE} = \frac{CJ}{CF} = \frac{1}{2}$, and since $\triangle GCJ$ and $\triangle ECF$ share a common angle at C, $\triangle GCJ$ is similar to $\triangle ECF$.

Therefore, $\angle GJC = \angle EFC = 90^\circ$, and $\frac{GJ}{EF} = \frac{CJ}{CF} = \frac{1}{2}$. Similarly, $\frac{HJ}{DF} = \frac{1}{2}$.

From $\angle GJC = 90^\circ$, we get that $GJHC$ is a triangular pyramid with base $\triangle GJH$ and height CJ.

Consider the volume of pyramid $GJHC$ divided by the volume of pyramid $EFDC$.

$$\begin{aligned}\frac{\text{Volume}(GJHC)}{\text{Volume}(EFDC)} &= \frac{\frac{1}{3} \times \frac{1}{2} \times GJ \times HJ \times CJ}{\frac{1}{3} \times \frac{1}{2} \times EF \times DF \times CF} \\ &= \frac{GJ}{EF} \times \frac{HJ}{DF} \times \frac{CJ}{CF} \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8}\end{aligned}$$

The solid $EFDHJG$ is obtained by removing pyramid $GJHC$ from pyramid $EFDC$. If we let W denote the volume of $EFDC$ and X denote the volume of $EFDHJG$, we have $X = W - \frac{1}{8}W = \frac{7}{8}W$.

By symmetry, the volume of $ACBHJG$ is also equal to $\frac{7}{8}W$. Therefore, the volume of the solid removed from the section of the cube in the diagram is $2 \times \frac{7}{8}W = \frac{7}{4}W$.

Four congruent solids are removed from the cube, so the total volume removed is $4 \times \frac{7}{4}W = 7W$.

The volume of the cube is $6 \times 6 \times 6 = 216$, and the volume of $EFDC$ is

$$W = \frac{1}{3} \times \frac{1}{2} \times EF \times DF \times CF = \frac{1}{6} \times 3 \times 3 \times 6 = 9$$

The volume of the remaining solid is $216 - 7 \times 9 = 153$, so the answer to the question is 53.

ANSWER: 53

25. The sum of the first $2k$ positive integers is

$$1 + 2 + 3 + \cdots + (2k-1) + 2k = \frac{2k(2k+1)}{2} = \frac{4k^2 + 2k}{2}$$

The sum of the k consecutive integers starting at $m \geq 1$ is

$$\begin{aligned}m + (m+1) + (m+2) + \cdots + (m+k-1) &= km + (1 + 2 + 3 + \cdots + (k-1)) \\ &= km + \frac{(k-1)k}{2} \\ &= \frac{k^2 + 2km - k}{2}\end{aligned}$$

Note that we must have $m \leq k+1$, otherwise $m + k - 1 > 2k$. Thus, the sum of the remaining integers is

$$\frac{4k^2 + 2k}{2} - \left(\frac{k^2 + 2km - k}{2}\right) = \frac{3k^2 - 2km + 3k}{2}$$

So, we are looking for positive integer pairs (k, m) for which $1 \leq m \leq k+1$ and

$$\frac{3k^2 - 2km + 3k}{2} = 819$$

Rearranging and factoring, this equation is

$$k(3k - 2m + 3) = 1638$$

Thus, we have that $(k, 3k-2m+3)$ is a divisor pair of 1638. We are assuming that $m \leq k+1$, and so $m < k+2$, from which it follows that $-2m > -2k-4$. Therefore, we have

$$\begin{aligned}(3k - 2m + 3) - k &= 2k - 2m + 3 \\ &> 2k + (-2k - 4) + 3 \\ &= -1\end{aligned}$$

Note that for an integer to be greater than -1, it must be at least 0. Therefore, $(3k-2m+3) \geq k$, and so in each divisor pair, $(k, 3k - m + 3)$, $3k - m + 3$ is the greater of the two. (1638 is not a perfect square, so the divisors will never be equal.) The *ordered* divisor pairs of 1638 are

$(1, 1638)$	$(7, 234)$	$(18, 91)$
$(2, 819)$	$(9, 182)$	$(21, 78)$
$(3, 546)$	$(13, 126)$	$(26, 63)$
$(6, 273)$	$(14, 117)$	$(39, 42)$

Now observe that $3(k+1) - (3k - 2m + 3) = 2m > 0$. Therefore, if we triple one more than the smaller divisor, the result must exceed the larger divisor. Of the divisor pairs above, only $(26, 63)$ and $(39, 42)$ have this property.
If $k = 26$ and $3k-2m+3 = 63$, then $m = 9$, and if $k = 39$ and $3k-2m+3 = 42$, then $m = 39$.

The only two possible values of k are $k = 26$ and $k = 39$, and their sum is $26 + 39 = 65$.

ANSWER: 65