



Problem of the Week

Problem E and Solution

Multiplying Tomatoes 2

Problem

Nia has four tomato plants, each with a different number of tomatoes on it. The product of the number of tomatoes on each plant is 24 300. The total number of tomatoes on the three plants with the most tomatoes is 43, and the total number of tomatoes on the three plants with the fewest tomatoes is 34. Determine all possibilities for the number of tomatoes on each plant.

Solution

This problem is asking us to find four distinct positive integers with a product of 24 300, such that the sum of the smallest three integers is 34 and the sum of the largest three integers is 43. We start by letting the four integers be a , b , c , and d . Since the sum of the smallest three integers is 34, it follows that $a + b + c = 34$. Since the sum of the largest three integers is 43, it follows that $b + c + d = 43$. Subtracting these two equations gives $d - a = 9$.

It would help if we had an upper bound for the smallest integer. Suppose $a = 12$. Then $d = 12 + 9 = 21$. The smallest product of four such distinct integers is $12 \times 13 \times 14 \times 21 = 45\,864$. Since this is larger than 24 300, it follows that $a < 12$.

Similarly, it would help to have a lower bound for the smallest integer. Suppose $a = 6$. Then $d = 6 + 9 = 15$. The largest product of four such distinct integers is $6 \times 13 \times 14 \times 15 = 16\,380$. Since this is smaller than 24 300, it follows that $a > 6$.

This leaves us with the following possibilities for (a, d) : $(7, 16)$, $(8, 17)$, $(9, 18)$, $(10, 19)$, or $(11, 20)$. Next we can factor 24 300 to obtain $24\,300 = 2^2 \times 3^5 \times 5^2$. From the factorization we can eliminate some of these pairs. Since 7, 17, 19, and 11 are not factors of 24 300, they cannot be any of the four integers. Thus, the only possibility left is $(a, d) = (9, 18)$.

Since $24\,300 = 2^2 \times 3^5 \times 5^2$, it follows that the middle two integers must have a product of $2 \times 3 \times 5^2$. The only possibilities between 9 and 18 are 10 and 15.

Therefore, there is only one possibility. The number of tomatoes on each plant is 9, 10, 15, and 18.