



Problem of the Week

Problem E and Solution

Math Test Math 3

Problem

Theo has had three tests in his math class so far. He does the following calculations using the average (mean) of some of his marks.

- First he calculates the average of his first and second test marks. Then he calculates the average of this value with his third test mark to get 74%.
- First he calculates the average of his second and third test marks. Then he calculates the average of this value with his first test mark to get 73.5%.
- First he calculates the average of his first and third test marks. Then he calculates the average of this value with his second test mark to get 76%.

Determine the average his three test marks.

Solution

Let a , b , and c represent the test marks, in order. We are looking for the average (mean) of a , b , and c , which is $\frac{a+b+c}{3}$.

The average of his first and second test marks is $\frac{a+b}{2}$. The average of this value with his third test mark is $\frac{\frac{a+b}{2}+c}{2}$. We know this is equal to 74. Thus,

$$\begin{aligned}\frac{\frac{a+b}{2}+c}{2} &= 74 \\ \frac{a+b}{2}+c &= 148 \\ a+b+2c &= 296\end{aligned}\tag{1}$$

Similarly, we can write the following two equations:

$$\begin{aligned}\frac{\frac{b+c}{2}+a}{2} &= 73.5 \\ \frac{b+c}{2}+a &= 147 \\ b+c+2a &= 294\end{aligned}\tag{2}$$
$$\begin{aligned}\frac{\frac{a+c}{2}+b}{2} &= 76 \\ \frac{a+c}{2}+b &= 152 \\ a+c+2b &= 304\end{aligned}\tag{3}$$



Next we add equations (1), (2), and (3):

$$(a + b + 2c) + (b + c + 2a) + (a + c + 2b) = 296 + 294 + 304$$

$$4a + 4b + 4c = 894$$

$$a + b + c = 223.5$$

$$\frac{a + b + c}{3} \approx 74.5$$

Therefore, the average of his three test marks is 74.5%.

EXTENSION: You may have noticed that the average of computed values 74, 73.5, and 76 is also equal to 74.5, the average of the test marks. In general, this will be the case for any three test results, when computed this way. Can you convince yourself of this?