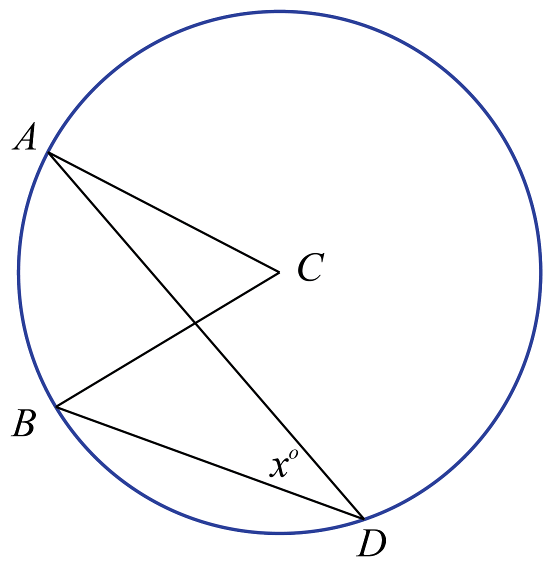
Problem of the Week
Problem E and Solution
Slice of an Arc

Problem

Points A, B, and D lie on the circumference of a circle with centre C.

If $\angle ADB = x^\circ$, then determine the measure of $\angle ACB$ in terms of x.

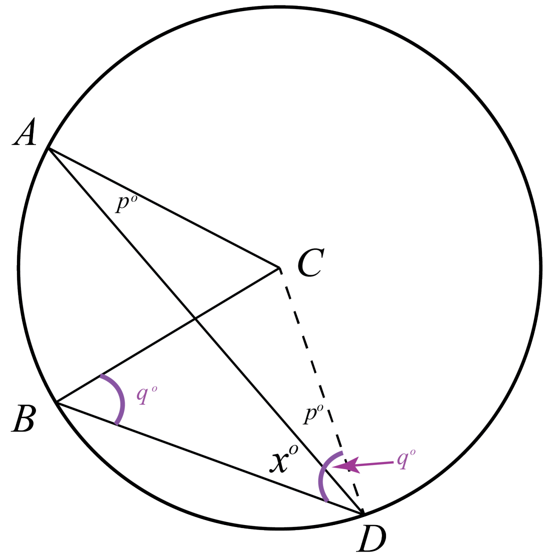


Solution

We construct radius CD. Let $\angle CAD = p^\circ$ and $\angle CBD = q^\circ$.

Since CA and CD are both radii of the circle, $CA = CD$. So $\triangle CAD$ is isosceles and $\angle CDA = \angle CAD = p^\circ$. Since the angles in a triangle add to 180°, $\angle ACD = (180 - 2p)^\circ$.

Since CB and CD are both radii of the circle, $CB = CD$. So $\triangle CBD$ is isosceles and $\angle CDB = \angle CBD = q^\circ$. Since the angles in a triangle add to 180°, $\angle BCD = (180 - 2q)^\circ$.



Now,

$$\begin{aligned}\angle ACB &= \angle ACD - \angle BCD \\ &= (180 - 2p)^\circ - (180 - 2q)^\circ \\ &= (2q - 2p)^\circ \\ &= 2(q - p)^\circ\end{aligned}$$

Since $\angle CDB = \angle CDA + \angle ADB$, we have $q = p + x$.

Thus, $\angle ACB = 2(q - p)^\circ = 2x^\circ$.

NOTE: In general, the angle inscribed at the centre of a circle is twice the size of the angle inscribed at the circumference by the same chord. That is, the angle inscribed by chord AB at the centre of the circle ($\angle ACB$) is double the angle inscribed by chord AB on the circumference ($\angle ADB$).