Problem of the Week

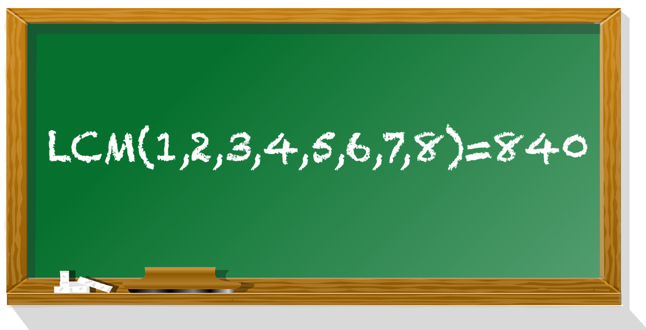
Problem E

Four More

For each positive integer n, $\text{LCM}(1, 2, \ldots, n)$ is the *least common multiple* of $1, 2, \ldots, n$. That is, the smallest positive integer divisible by each of $1, 2, \ldots, n$.

Determine all positive integers n, with $1 \leq n \leq 100$ such that

$$\text{LCM}(1, 2, \ldots, n) = \text{LCM}(1, 2, \ldots, n + 4)$$



NOTE: In solving this problem, it might be helpful to know that we can calculate the LCM of a set of positive integers by

- determining the prime factorization of each integer in the set,
- determining the list of prime numbers that occur in these prime factorizations,
- determining the highest power of each prime number from this list that occurs in the prime factorizations, and
- multiplying these highest powers together.

For example, $\text{LCM}(1, 2, 3, 4, 5, 6, 7, 8) = 2^3 \cdot 3^1 \cdot 5^1 \cdot 7^1 = 840$, since the prime factorizations of 2, 3, 4, 5, 6, 7, and 8 are 2, 3, 2^2, 5, $2 \cdot 3$, 7, and 2^3, respectively.