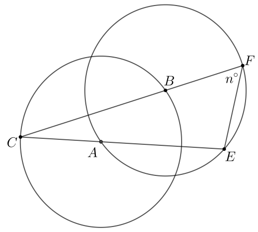


Problem of the Week
Problem E and Solution
Circles and Corners Curiosity

Problem

Two circles, with centres A and B, intersect so that A lies on the circle with centre B, and B lies on the circle with centre A. Point C lies on the circle with centre A and points E and F lie on the circle with centre B so that CAE and CBF are straight line segments.

If $\angle CFE = n^\circ$, with $0 < n < 90$, determine the measure of $\angle FCE$ in terms of n.

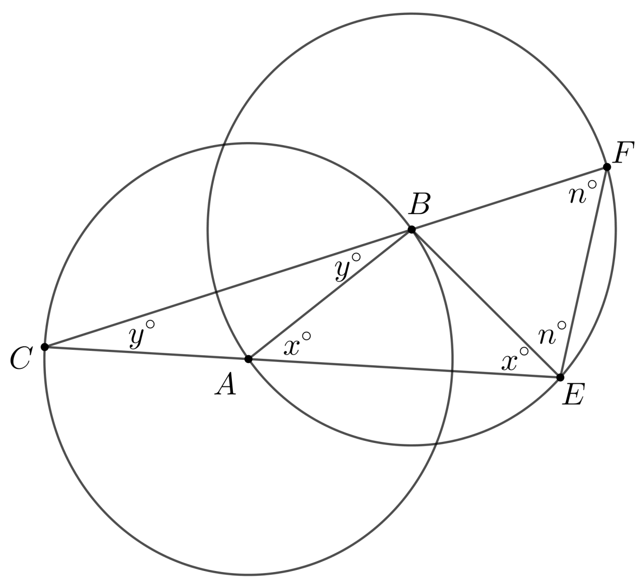
Solution

Draw in AB and BE. Points A, E, and F lie on the circumference of the circle with centre B. Therefore, $BA = BE = BF$. Points B and C lie on the circle with centre A, thus $AB = AC$.

Let $\angle BEA = x^\circ$. Since $BA = BE$, $\triangle BAE$ is isosceles, and so $\angle BAE = \angle BEA = x^\circ$.

Let $\angle ABC = y^\circ$. Since $AB = AC$, $\triangle ABC$ is isosceles, and so $\angle BCA = \angle ABC = y^\circ$.

Also, since $BE = BF$, $\triangle BEF$ is isosceles, and so $\angle BEF = \angle BFE = \angle CFE = n^\circ$.



$\angle BAE$ is an exterior angle to $\triangle ABC$. By the Exterior Angle Theorem for triangles, $\angle BAE = \angle BCA + \angle ABC$. Therefore, $x = 2y$.

In $\triangle CEF$, since the angles in a triangle sum to 180°, we have $\angle FCE + \angle CFE + \angle CEF = 180^\circ$. Since $\angle FCE = \angle BCA = y^\circ$, $\angle CFE = n^\circ$, and $\angle CEF = \angle BEA + \angle BEF = x^\circ + n^\circ$, we have $y^\circ + n^\circ + x^\circ + n^\circ = 180^\circ$. Thus, $y + x + 2n = 180$.

Since $x = 2y$, we have $y + 2y + 2n = 180$, which simplifies to $3y = 180 - 2n$ or $y = 60 - \frac{2}{3}n$.

Therefore, $\angle FCE = y^\circ = \left(60 - \frac{2}{3}n\right)^\circ$.