Problem of the Week
Problem E and Solution
A Heavy Problem

Problem

Four boxes: one blue, one green, one pink, and one yellow, each have a different mass. The mass of each box, in grams, is a positive integer. Inkeri does not know the individual masses of the boxes, but she knows the combined mass of the pink, blue, and green boxes is 13 grams. She also knows the combined mass of the blue, green, and yellow boxes is 17 grams, and the combined mass of the pink, green, and yellow boxes is 19 grams. Determine all possibilities for the individual masses of the boxes.

Solution

Let b, g, p, and y represent the masses, in grams, of the blue, green, pink, and yellow boxes, respectively. It is given that $b \neq g \neq p \neq y$ and each of the masses is a positive integer. We also know the following:

$$p + b + g = 13 \quad (1)$$
$$b + g + y = 17 \quad (2)$$
$$p + g + y = 19 \quad (3)$$

Subtracting (2) − (1) gives $y - p = 4$, or equivalently, $y = p + 4$.

Subtracting (3) − (2) gives $p - b = 2$, or equivalently, $b = p - 2$.

From here we proceed with two different approaches.

Solution 1

From (1), we obtain $g = 13 - p - b$. Since p, b, g, and y are all different positive integers, we can look at the possible values of p and calculate the values of b, g, and y in each case. Then we can determine if it is a possible set of masses. This is summarized in the following table.

Mass of Pink Box, p	Mass of Blue Box, $b = p - 2$	Mass of Green Box, $g = 13 - p - b$	Mass of Yellow Box, $y = p + 4$	Possible?
1	−1	13	5	No, $b < 1$
2	0	11	6	No, $b < 1$
3	1	9	7	Yes
4	2	7	8	Yes
5	3	5	9	No, $p = g$
6	4	3	10	Yes
7	5	1	11	Yes

We can stop here, because if $p > 7$, then $g < 1$, which is not valid. Therefore, there are four different possibilities for the masses of the blue, green, pink, and yellow boxes, respectively. They are:

$$(b, g, p, y) = (1, 9, 3, 7),\ (2, 7, 4, 8),\ (4, 3, 6, 10),\ (5, 1, 7, 11)$$

Solution 2

This solution is similar to Solution 1. The key difference is that in this solution, we find expressions for b, g, and y, in terms of p. We then determine the smallest and largest possible values for p.

We substitute $y = p + 4$ and $b = p - 2$ into (2) to get an expression for g in terms of p.

$$\begin{aligned} b + g + y &= 17 \\ (p - 2) + g + (p + 4) &= 17 \\ 2p + g + 2 &= 17 \\ g &= 15 - 2p \end{aligned}$$

Since $b = p - 2$ and b is a positive integer, the smallest positive integer value for p will be 3. Otherwise $b < 1$. Since $g = 15 - 2p$ and g is a positive integer, the largest positive integer value for p will be 7. Otherwise $g < 1$. Therefore, the only possible values for p are 3, 4, 5, 6, and 7.

We will now look at each possible value of p, calculate the values of b, g, and y in each case, and determine if it is a possible set of masses.

Mass of Pink Box, p	Mass of Blue Box, $b = p - 2$	Mass of Green Box, $g = 15 - 2p$	Mass of Yellow Box, $y = p + 4$	Possible?
3	1	9	7	Yes
4	2	7	8	Yes
5	3	5	9	No, $p = g$
6	4	3	10	Yes
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Therefore, there are four different possibilities for the masses of the blue, green, pink, and yellow boxes, respectively. They are:

$$(b, g, p, y) = (1, 9, 3, 7),\ (2, 7, 4, 8),\ (4, 3, 6, 10),\ (5, 1, 7, 11)$$