Problem of the Week
Problem E and Solution
Spinning Red

Problem

A spinner is divided into 15 equal sections. Each section is coloured either red or yellow. An arrow is attached to the centre of the spinner. Jamal spins the arrow two times. If there is a 64% chance of landing on red in at least one of the two spins, how many red sections are there?

NOTE: You may use the following fact from probability theory: If the probability of event A occurring is a, the probability of event B occurring is b, and the events are not dependent on each other, then the probability of both events occurring is $a \times b$.

Solution

Solution 1

Let n be the number of yellow sections. Therefore, for each spin, the probability the spinner lands on a yellow section (and therefore does not land on a red section) is $\frac{n}{15}$. Since the result of each spin does not depend on the previous spin,

$$\begin{aligned} P(\text{not red on either spin}) &= P(\text{not red on first spin}) \times P(\text{not red on second spin}) \\ &= \frac{n}{15} \times \frac{n}{15} \\ &= \left(\frac{n}{15}\right)^2 \end{aligned}$$

The probability of landing on at least one red in two spins is 0.64. So, the probability of not landing on red in either of the two spins is $1 - 0.64 = 0.36$. That is,

$$\left(\frac{n}{15}\right)^2 = 0.36$$

Since $n \geq 0$, this simplifies to $\frac{n}{15} = 0.6$, and so $n = 9$.

Since n is the number of yellow sections, there are $15 - 9 = 6$ red sections.

Solution 2

Let r be the number of red sections. Therefore, the number of yellow sections is $15 - r$. Also, for each spin, the probability the spinner lands on a red section is $\frac{r}{15}$ and the probability the spinner lands on a yellow section is $\frac{15-r}{15}$.

If Jamal lands on red in at least one spin, then he may land red on the first spin only, red on the second spin only, or red on both spins.

If Jamal lands on red on his first spin only, then his second spin must land on yellow. Since the results of each spin do not depend on each other, the probability that he spins red on his first spin and yellow on his second spin is $\frac{r}{15} \times \frac{15-r}{15} = \frac{r(15-r)}{15^2}$.

If Jamal lands on red on his second spin only, then his first spin must land on yellow. Thus, the probability Jamal spins yellow on his first spin and red on his second spin is $\frac{15-r}{15} \times \frac{r}{15} = \frac{r(15-r)}{15^2}$.

The probability that Jamal lands on red on his first spin and again on his second spin is $\frac{r}{15} \times \frac{r}{15} = \frac{r^2}{15^2}$.

The probability of getting at least one red in the two spins is equal to the probability that he lands red on his first spin only, plus the probability that he lands red on his second spin only, plus the probability that he lands red on both spins. That is,

$$\frac{r(15-r)}{15^2} + \frac{r(15-r)}{15^2} + \frac{r^2}{15^2} = 0.64$$

Multiplying both sides by 15^2 gives

$$r(15-r) + r(15-r) + r^2 = 144$$

This simplifies to $r^2 - 30r + 144 = 0$. By factoring, we obtain $(r-6)(r-24) = 0$. Therefore, $r = 6$ or $r = 24$.

Since the spinner has only 15 sections, we must have $r \leq 15$. Thus, the only solution is $r = 6$. That is, there are 6 red sections.

Solution 3

Let r be the number of red sections. Therefore, the number of yellow sections is $15 - r$. Also, for each spin, the probability the spinner lands on a red section is $\frac{r}{15}$ and the probability the spinner lands on a yellow section is $\frac{15-r}{15}$.

If Jamal lands on red in at least one spin, then the first red occurs on his first spin or on his second spin.

If Jamal lands on the first red on his first spin, then on his first spin he spins a red, and on his second spin he spins any colour. Since the results of each spin do not depend on each other, the probability that the first red occurs on his first spin is $\frac{r}{15} \times \frac{15}{15} = \frac{r}{15}$.

If Jamal lands on the first red on the second spin, then on his first spin he spins a yellow, and on his second spin he spins a red. The probability of this is $\frac{15-r}{15} \times \frac{r}{15} = \frac{(15-r)r}{15^2}$.

The probability of spinning at least one red on the two spins is equal to the probability that he lands the first red on his first spin, plus the probability that he lands the first red on his second spin. That is,

$$\begin{aligned} \frac{r}{15} + \frac{(15-r)r}{15^2} &= 0.64 \\ 15r + (15-r)r &= 144 \\ r^2 - 30r + 144 &= 0 \end{aligned}$$

Using the quadratic formula, we find $r = \dfrac{30 \pm \sqrt{30^2 - 4(1)(144)}}{2} = \dfrac{30 \pm 18}{2} = 24, 6.$

Since the spinner has only 15 sections, we must have $r \leq 15$. Thus, the only solution is $r = 6$. That is, there are 6 red sections.