# Problem of the Week
Problem E and Solution
Free Time

Problem

Pearl coordinates volunteers to help collect and sort donations at the food bank on Mondays, Wednesdays, and Fridays. Of her volunteers, 50% are available on Mondays, 80% are available on Wednesdays, and 90% are available on Fridays. A total of 18 volunteers are available on all three days, and all other volunteers are available on exactly two of the three days. How many volunteers are there in total?

Solution

Let a be the number of volunteers who are available on Mondays and Wednesdays, but not Fridays. Let b be the number of volunteers who are available on Mondays and Fridays, but not Wednesdays. Let c be the number of volunteers who are available on Wednesdays and Fridays, but not Mondays. We note that none of the volunteers are available on zero or only one of the three days, and that 18 volunteers are available on all three days. We summarize this information in the following Venn diagram.
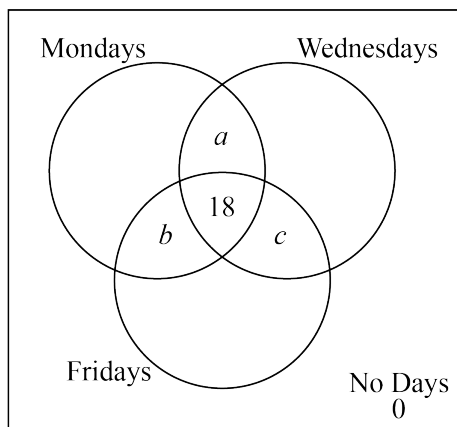
Let n be the total number of volunteers. Then, $n = a + b + c + 18$. From the given information,

- 50% of the volunteers are available on Mondays, so $0.5n = a + b + 18$.

- 80% of the volunteers are available on Wednesdays, so $0.8n = a + c + 18$.

- 90% of the volunteers are available on Fridays, so $0.9n = b + c + 18$.

Since $n = a + b + c + 18$, it follows that $2n = 2a + 2b + 2c + 36$. Then,

$$\begin{aligned}
2n &= 2a + 2b + 2c + 36 \\
&= (a + b + 18) + (a + c + 18) + (b + c) \\
&= 0.5n + 0.8n + 0.9n - 18 \\
&= 2.2n - 18 \\
18 &= 0.2n \\
n &= 90
\end{aligned}$$

Thus, there are 90 volunteers in total.