



Problem of the Week

Problem D and Solution

Multiplying Tomatoes 1

Problem

Jamal grew four tomato plants last summer. The product of the total number of tomatoes each plant produced is 40 392. The plant that produced the most tomatoes produced exactly 50 more tomatoes than the plant that produced the fewest tomatoes. If the plant that produced the fewest tomatoes produced fewer than 10 tomatoes, determine all possibilities for the number of tomatoes each plant produced.

Solution

This problem is asking us to find four positive integers with a product of 40 392, where the difference between the smallest and largest integer is 50, and the smallest integer is less than 10. We can start by factoring 40 392. This gives $40\,392 = 1 \times 2^3 \times 3^3 \times 11 \times 17$. We have included 1 since it could be one of the positive integers.

Since the smallest integer is less than 10, from the factorization we can see that it could be 1, 2, 3, 4, 6, 8, or 9. We will consider these cases.

- If the smallest integer is 1, then the largest integer must be 51. Since $51 = 3 \times 17$, it follows that 51 is a factor of 40 392. The product of the other two integers must then be $2^3 \times 3^2 \times 11 = 792$.

Thus, we are looking for two numbers, both less than 51, that multiply to 792. The factor pairs of 792 with both factors less than 51 are: 18 and 44, 22 and 36, and 24 and 33.

Therefore, the four integers could be

- 1, 18, 44, and 51
- 1, 22, 36, and 51
- 1, 24, 33, and 51

- If the smallest integer is 2, then the largest integer must be 52. However 52 is *not* a factor of 40 392, so this is not a possibility.
- If the smallest integer is 3, then the largest integer must be 53. However 53 is *not* a factor of 40 392, so this is not a possibility.
- If the smallest integer is 4, then the largest integer must be 54. Since $54 = 2 \times 3^3$, it follows that 54 is a factor of 40 392. The product of the other



two integers must then be 11×17 . Thus, the four integers could be 4, 11, 17, and 54.

- If the smallest integer is 6, then the largest integer must be 56. However 56 is *not* a factor of 40 392, so this is not a possibility.
- If the smallest integer is 8, then the largest integer must be 58. However 58 is *not* a factor of 40 392, so this is not a possibility.
- If the smallest integer is 9, then the largest integer must be 59. However 59 is *not* a factor of 40 392, so this is not a possibility.

Therefore, there are four possibilities for the number of tomatoes each plant produced. They are as follows:

- 1, 18, 44, and 51
- 1, 22, 36, and 51
- 1, 24, 33, and 51
- 4, 11, 17, and 54