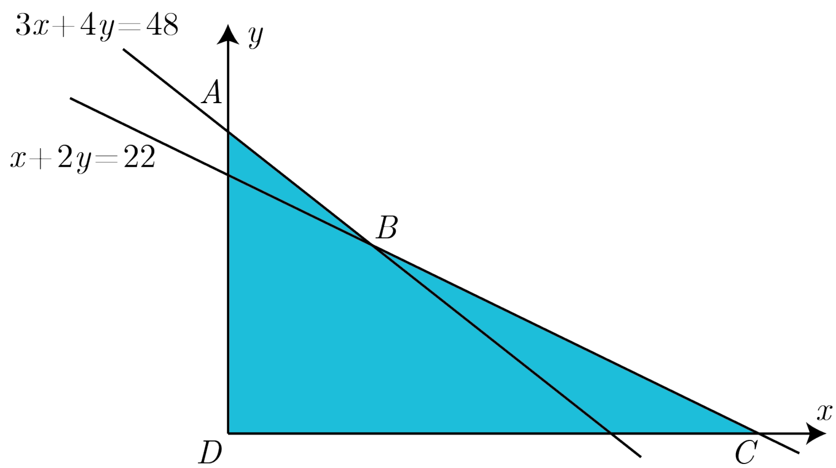
Problem of the Week

Problem D and Solution

Shading the Quad

Problem

Peggie draws the line $3x + 4y = 48$ and labels its intersection with the y-axis A. Derrick then draws the line $x + 2y = 22$ on the same set of axes and labels its intersection with Peggie's line B. Derrick then labels the intersection of his line with the x-axis C. Peggie then labels the origin D and shades in quadrilateral $ABCD$, as shown.



Determine the area of quadrilateral $ABCD$.

Solution

We start by finding the coordinates of points A, B, and C. Since D is the origin, its coordinates are $(0, 0)$.

To determine the coordinates of point A, we need to find the y-intercept of the line $3x + 4y = 48$. To find this, let $x = 0$. Then $4y = 48$, so $y = 12$. Thus, the coordinates of A are $(0, 12)$.

To determine the coordinates of point C, we need to find the x-intercept of the line $x + 2y = 22$. To find this, let $y = 0$. Then $x = 22$. Thus, the coordinates of C are $(22, 0)$.

Point B is the point of intersection of the lines $3x + 4y = 48$ and $x + 2y = 22$. To find this, we solve the following system of equations.

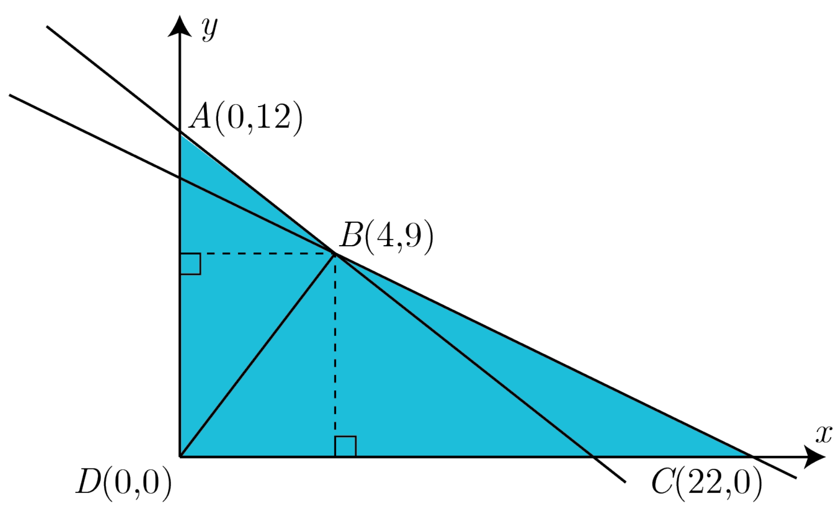
$$3x + 4y = 48 \qquad (1)$$
$$x + 2y = 22 \qquad (2)$$

From equation (2), we solve for x to obtain $x = 22 - 2y$. Substituting into equation (1), we have

$$\begin{aligned} 3(22-2y)+4y &= 48 \\ 66-6y+4y &= 48 \\ -2y &= -18 \\ y &= 9 \end{aligned}$$

Then substituting back into $x = 22 - 2y$, we obtain $x = 22 - 2(9) = 4$. Thus, the coordinates of B are $(4, 9)$.

We now draw line segment BD to divide quadrilateral $ABCD$ into two triangles: $\triangle ABD$ and $\triangle CBD$ as shown.



$\triangle ABD$ has a base of 12 and a height of 4, and $\triangle CBD$ has a base of 22 and a height of 9. Thus,

$$\begin{aligned} \text{Area } ABCD &= \text{Area } \triangle ABD + \text{Area } \triangle CBD \\ &= \frac{1}{2}(12)(4) + \frac{1}{2}(22)(9) \\ &= 24 + 99 \\ &= 123 \end{aligned}$$

Therefore, the area of quadrilateral $ABCD$ is 123 units^2.