Problem of the Week
Problem D and Solution
Math Test Math 2

Problem

Milana has had six tests in her math class so far. She knows the average (mean) of some of her marks.

- The average of her first and second test marks is 85%.
- The average of her second and third test marks is 82%.
- The average of her third and fourth test marks is 74%.
- The average of her fourth and fifth test marks is 79%.
- The average of her fifth and sixth test marks is 80%.

Determine the average of her first and sixth test marks.

Solution

Let a, b, c, d, e, and f represent Milana's six test marks, in order. We are looking for the average of a and f. We know the following.

- The average of her first and second test marks is 85, so $\frac{a+b}{2} = 85$, which gives $a+b = 170$.
- The average of her second and third test marks is 82, so $\frac{b+c}{2} = 82$, which gives $b+c = 164$.
- The average of her third and fourth test marks is 74, so $\frac{c+d}{2} = 74$, which gives $c+d = 148$.
- The average of her fourth and fifth test marks is 79, so $\frac{d+e}{2} = 79$, which gives $d+e = 158$.
- The average of her fifth and sixth test marks is 80, so $\frac{e+f}{2} = 80$, which gives $e+f = 160$.

Adding these six equations gives the following:

$$\begin{aligned}
(a+b)+(b+c)+(c+d)+(d+e)+(e+f) &= 170+164+148+158+160 \\
a+2b+2c+2d+2e+f &= 800 \\
a+2(b+c)+2(d+e)+f &= 800 \\
a+2(164)+2(158)+f &= 800 \\
a+328+316+f &= 800 \\
a+f &= 156 \\
\frac{a+f}{2} &= 78
\end{aligned}$$

Therefore, the average of Milana's first and sixth test marks is 78%.