Problem of the Week
Problem D and Solution
Sports Day

Problem

There were 500 students at a sports day camp. For the morning session, each student could choose between soccer or basketball. For the afternoon session, each student could choose between hockey or frisbee. Each student had to choose exactly one sport per session.

The following information is known about the sessions:

- 32% of the students chose basketball,
- $\frac{1}{5}$ of the students who chose hockey also chose basketball, and
- of the students who chose basketball, 50 more chose frisbee than hockey.

Determine the percentage of students who choose both soccer and frisbee.

Solution

There were 500 students in total and 32% of them chose basketball. Thus, $0.32 \times 500 = 160$ students chose basketball. Since students had to choose either basketball or soccer, it follows that the remaining $500 - 160 = 340$ students chose soccer. The table shows the information we have determined so far about the number of students who chose each sport.

	Soccer	**Basketball**	**Total**
Hockey			
Frisbee			
Total	340	160	500

We know that of the 160 students who chose basketball, 50 more chose frisbee than hockey. So, if h students chose basketball and hockey, then $h + 50$ students chose basketball and frisbee. Since $h + (h + 50) = 160$, it follows that $2h + 50 = 160$, so $2h = 110$, which gives $h = 55$. Thus, 55 of the students who chose basketball also chose hockey. Then the remaining $160 - 55 = 105$ students who chose basketball chose frisbee. We can now update our table accordingly.

	Soccer	**Basketball**	**Total**
Hockey		55	
Frisbee		105	
Total	340	160	500

Next, we know that $\frac{1}{5}$ of the students who chose hockey also chose basketball. Since 55 students chose hockey and basketball, this tells us that 55 is $\frac{1}{5}$ of the number of students who chose hockey. Thus, the number of students who chose hockey is $5 \times 55 = 275$. From here we can determine that the number of students who chose soccer and hockey is $275 - 55 = 220$. Then the number of students who chose soccer and frisbee is $340 - 220 = 120$. Though not required, the total number of students who chose frisbee is $120 + 105 = 225$. Our completed table is shown.

	Soccer	**Basketball**	**Total**
Hockey	220	55	275
Frisbee	120	105	225
Total	340	160	500

Finally, we can determine that the percentage of students who choose both soccer and frisbee is $\frac{120}{500} \times 100\% = 24\%$.