Problem of the Week
Problem D and Solution
A Sweet Mix

Problem

Mariana is buying cinnamon sugar, which is a mix made of cinnamon and sugar. There are two kinds available; one is in a red package and the other is in a blue package, but both packages have the same mass. In the red package, the ratio of cinnamon to sugar is $1 : 3$, by mass. In the blue package, the ratio of cinnamon to sugar is $1 : 4$, by mass. Mariana buys one red package and two blue packages, then mixes them together to create a new cinnamon sugar mix. Determine the ratio of cinnamon to sugar in Mariana's new cinnamon sugar mix.

Solution

Solution 1

Let x represent the mass of each package of cinnamon sugar. Then the total mass of one red package of mix is x and the total mass of two blue packages of mix is $2x$.

Since the ratio of cinnamon to sugar in the red package, by mass, is $1 : 3$, then $\frac{1}{4}$ of the mass of the red package mix is cinnamon. It follows that the total mass of cinnamon in the red package mix is $\frac{1}{4}x$ and the total mass of sugar in the red package mix is $\frac{3}{4}x$.

Similarly, since the ratio of cinnamon to sugar in the blue package, by mass, is $1 : 4$, then $\frac{1}{5}$ of the mass of the blue package mix is cinnamon. It follows that the total mass of cinnamon in two blue packages of mix is $\frac{1}{5}(2x) = \frac{2}{5}x$ and the total mass of sugar in two blue packages of mix is $\frac{4}{5}(2x) = \frac{8}{5}x$.

When the packages are mixed to create the new cinnamon sugar mix, we obtain the following:

- Total mass of cinnamon: $\frac{1}{4}x + \frac{2}{5}x = \frac{5}{20}x + \frac{8}{20}x = \frac{13}{20}x$
- Total mass of sugar: $\frac{3}{4}x + \frac{8}{5}x = \frac{15}{20}x + \frac{32}{20}x = \frac{47}{20}x$

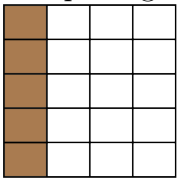
Thus, the ratio of cinnamon to sugar is $\frac{13}{20}x : \frac{47}{20}x = 13 : 47$.

Solution 2

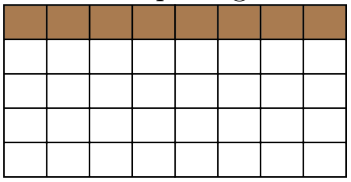
Let $\frac{1}{20}$ of each package be a unit of mass. Since the ratio of cinnamon to sugar in the red package, by mass, is $1 : 3$, then $\frac{1}{4}$ of the package, or 5 units of mass are cinnamon. Then the remaining 15 units are sugar.

Since there are two blue packages, then there are 40 units of mass in total of the blue package mix. Since the ratio of cinnamon to sugar in the blue package, by mass, is $1 : 4$, then $\frac{1}{5}$ of the mix, or 8 units of mass are cinnamon. Then the remaining 32 units are sugar. This is illustrated in the following diagrams, where the units of cinnamon are shaded.

red package



blue package



When the packages are combined, there is a total of 60 units of mass. Of these, $5 + 8 = 13$ are cinnamon and $15 + 32 = 47$ are sugar. Thus, the ratio of cinnamon to sugar is $13 : 47$.