



Problem of the Week

Problem D and Solution

Difference Detective

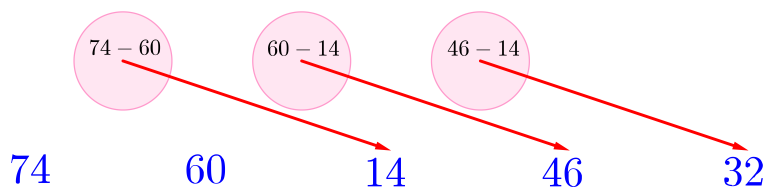
Problem

The *non-negative difference* between two numbers a and b is $b - a$ or $a - b$, whichever is greater than or equal to 0. For example, the non-negative difference between 24 and 64 is 40.

Consider the sequence with first term 74, second term 60, and each term after equal to the non-negative difference between the previous two terms. That is, the third term is $74 - 60 = 14$, the fourth term is $60 - 14 = 46$, and the fifth term is $46 - 14 = 32$. So, the first five terms in the sequence are:

$$74, 60, 14, 46, 32$$

Determine the sum of the first 1306 terms in the sequence.



Solution

We start by generating more terms in the sequence in an attempt to find a pattern.

Using the rule for creating the sequence, we determine that the first 23 terms are

$$74, 60, 14, 46, 32, 14, 18, 4, 14, 10, 4, 6, 2, 4, 2, 2, 0, 2, 2, 0, 2, 2, 0$$

The first 14 terms in the sequence have no apparent pattern. Afterwards, the terms repeat 2, 2, 0. Since each term is determined by the previous two, this sequence of 2, 2, 0 will repeat indefinitely.

Thus, the first 14 terms in the sequence are followed by n groups of 2, 2, 0. What is the value of n ? We want a total of 1306 terms. If we remove the first 14 terms, we require $1306 - 14 = 1292$ more terms. If we divide 1292 by 3 we are able to determine how many complete sequences of 2, 2, 0 are needed. Since $1292 \div 3 = 430\frac{2}{3}$, there will be 430 complete sequences of 2, 2, 0 followed by 2, 2.

The required sum is the sum of the first 14 terms plus $430 \times (2 + 2 + 0) + 2 + 2$. The sum of the first 14 terms is 302. Thus, the sum of the first 1306 terms in the sequence is $302 + 430 \times 4 + 4 = 2026$.

Therefore, the sum of the first 1306 terms in the sequence is 2026.