Problem of the Week
Problem D and Solution
It's Still Five

Problem

When an integer, N, is divided by 10, 11, or 12, the remainder is 5. If $N > 5$, what is the smallest possible value of N?

$$10\overline{)N} \qquad 11\overline{)N} \qquad 12\overline{)N}$$

Solution

Solution 1

When N is divided by 10, 11, or 12, the remainder is 5. This means that $M = N - 5$ is divisible by each of 10, 11, and 12. Since M is divisible by each of 10, 11, and 12, then M is divisible by the least common multiple of 10, 11, and 12.

Since $10 = 2 \times 5$, $12 = 2^2 \times 3$, and 11 is prime, then to find the least common multiple, we calculate the product of the highest powers of each of the prime factors that occur in the given numbers. It follows that the least common multiple of 10, 11, and 12 is $2^2 \times 3 \times 5 \times 11 = 660$.

Since $N > 5$ and $M = N - 5$, we can conclude that $M > 0$. Since M is divisible by 660, then the smallest possible value for M is 660. Then $N = 660 + 5 = 665$.

Solution 2

When N is divided by 10, 11, or 12, the remainder is 5. This means that $M = N - 5$ is divisible by each of 10, 11, and 12. Since M is divisible by 10 and 11, then M must be divisible by the least common multiple of 10 and 11, which is 110. We test the first few multiples of 110 until we obtain one that is divisible by 12.

The integers 110, 220, 330, 440, and 550 are not divisible by 12, but 660 is. Therefore, M is divisible by 660. Since $N > 5$ and $M = N - 5$, we can conclude that $M > 0$. Therefore, the smallest possible value for M is 660. Then $N = 660 + 5 = 665$.