Problem of the Week
Problem D and Solution
A Close Race

Problem

Joshua and Berhino are two Olympic long distance runners. Their qualifying times were equal ten weeks prior to the Olympics. Two weeks after qualifying, Joshua got an injury and his time increased by 3.2%, but he was able to decrease that time by 8.1% before the Olympics. Berhino's time decreased by 2% and then again by r% over the same time period.

If they finished the Olympic race with the same time, determine the value of r, rounded to the nearest tenth.

Solution

Let t be Joshua's Olympic qualifying time. Since Joshua and Berhino had the same qualifying time, then t is also the qualifying time for Berhino.

Due to injury, Joshua's time increased by 3.2% two weeks after qualifying, so his time was

$$t + \frac{3.2}{100}t = \left(1 + \frac{3.2}{100}\right)t = 1.032t$$

He was able to decrease that time by 8.1% before the Olympics, so the new time is

$$1.032t - \left(\frac{8.1}{100}\right)(1.032t) = \left(1 - \frac{8.1}{100}\right)(1.032t) = 0.919(1.032t) = 0.948408t$$

Berhino's time first decreased by 2.0%, so his time was

$$t - \frac{2.0}{100}t = \left(1 - \frac{2.0}{100}\right)t = 0.98t$$

His time then further decreased by r%, so his time was

$$0.98t - \frac{r}{100}(0.98t) = 0.98t - \frac{0.98rt}{100}$$

Since they finished the Olympic race with the same time,

$$\begin{aligned} 0.948408t &= 0.98t - \frac{0.98rt}{100} \\ 0.948408t - 0.98t &= -\frac{0.98rt}{100} \\ 0.031592t &= \frac{0.98rt}{100} \end{aligned}$$

Dividing both sides by $t > 0$, we have

$$\begin{aligned} 0.031592 &= \frac{0.98r}{100} \\ 3.1592 &= 0.98r \\ r &\approx 3.2 \end{aligned}$$

Therefore, rounded to the nearest tenth, $r \approx 3.2$.