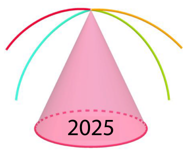

Problem of the Week

Problem D and Solution

Welcome to a New Year!

Problem

5^3 is a *power* with *base* 5 and *exponent* 3.

5^3 means $5 \times 5 \times 5$ and is equal to 125 when expressed as an integer.

When $8^{674} \times 5^{2025}$ is expressed as an integer, how many digits are in the product?

Solution

An immediate temptation might be to reach for a calculator. In this case, basic calculator technology will let you down. We will solve this problem using our knowledge of powers and corresponding power laws.

$$\begin{aligned} 8^{674} \times 5^{2025} &= \left((2^3)^{674}\right) \times 5^{2025} \\ &= 2^{3\times 674} \times 5^{2025} \\ &= 2^{2022} \times 5^{2025} \\ &= 2^{2022} \times 5^{2022} \times 5^3 \\ &= (2 \times 5)^{2022} \times 125 \\ &= 10^{2022} \times 125 \end{aligned}$$

But 10^{2022} is the number 1 followed by 2022 zeroes. When we multiply this number by the three-digit number 125, we obtain the number 125 followed by 2022 zeroes. Therefore, $8^{674} \times 5^{2025}$ has $2022 + 3 = 2025$ digits. Happy New Year again!