



Problem of the Week Problem D and Solution Triangled

Problem

Consider the following diagram.



If AE = 60, DE = 8, and CD = 17, determine the length of BC.

Solution

Since $\triangle CDE$ is right-angled, we use the Pythagorean theorem to solve for CE.

$$CE^2 = CD^2 - DE^2 = 17^2 - 8^2 = 225$$

Thus $CE = \sqrt{225} = 15$, since CE > 0. The diagram is now updated with all the lengths we know so far.



Since AC + CE = AE, it follows that AC = 60 - 15 = 45. Since $\angle ACB$ and $\angle DCE$ are opposite angles, then $\angle ACB = \angle DCE$. We also know that $\angle CAB = \angle CED = 90^{\circ}$, so we can conclude that $\triangle ABC \sim \triangle EDC$. Then,

$$\frac{BC}{AC} = \frac{CD}{CE}$$
$$\frac{BC}{45} = \frac{17}{15}$$
$$BC = 51$$

Thus, the length of BC is 51.