

Problem of the Week

Problem D and Solution

Might I Win Every Time?

Problem

In this two-player game, the vertices of a regular hexagon are each covered by a circle.

On a turn, a player may either initial one circle or initial two adjacent circles. (The two adjacent circles must be directly connected by an outside edge of the hexagon.) Players alternate turns. The player initialing the last circle or last pair of adjacent circles is the winner.

Two players, Cameron and Dale, play the game with Cameron going first.

One of the players, Cameron or Dale, can always win the game no matter what moves the other player makes. Describe which player can always win and the winning strategy for that player.

Solution

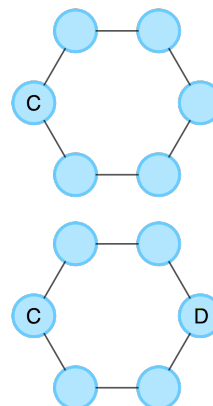
We will show that Dale can always win the game, and will describe Dale's winning strategy.

There are two types of possible first moves for Cameron: Cameron could initial exactly one blank circle, or Cameron could initial two adjacent blank circles.

- **Case 1:** Cameron initials exactly one blank circle on his first move.

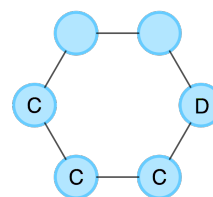
We can rotate the hexagon without changing the game, so we will assume that Cameron initials the circle on the left with a 'C', as shown.

In this case, Dale should initial the circle on the opposite side of the hexagon with a 'D'.



There are now two types of possible second moves for Cameron: Cameron could initial exactly one blank circle, or Cameron could initial two adjacent blank circles.

Suppose Cameron initials two adjacent circles, either at the top or bottom, on his second turn. The situation where he initials the bottom two circles is shown.





Dale should then initial the two remaining adjacent circles on the opposite side to win the game.

Instead, suppose Cameron initials only one circle on his second turn. The situation where he initials the bottom left circle is shown.

Dale should then initial one of the two adjacent unmarked circles on the other side of the hexagon. The situation where he initials the top left circle is shown.

Now there are two non-adjacent unmarked circles left. On his third turn, Cameron must initial one of these unmarked circles. Dale then wins the game by initialing the last unmarked circle.

- **Case 2:** Cameron initials two adjacent blank circles on his first move.

Again, we can rotate the hexagon without changing the game, so we will assume that Cameron initials the two unmarked circles at the top.

In this case, Dale should initial the two adjacent unmarked circles at the bottom.

Now there are two non-adjacent unmarked circles left, and Cameron must initial exactly one of these unmarked circles. Dale then initials the last unmarked circle to win the game.

We have considered all of the possible cases and have shown that Dale has a winning strategy in each case. Dale should “copy” Cameron by initialing the same number of unmarked circles as Cameron, but on the opposite side of the hexagon. If Dale follows this strategy, Dale will always win.

FOR FURTHER THOUGHT:

Without changing the rules of the game, how would the strategy change if instead of a hexagon with 6 circles, there were a heptagon with 7 circles?

