



Problem of the Week

Problem D and Solution

Birthday Cake Trouble

Problem

Harri is the manager of a pet store. For his birthday, his employees decided to surprise him with a birthday cake. The cake cost \$65.40 and everyone agreed to split the cost evenly. However, when it came time to collect the money, three of the employees were nowhere to be found. This meant that everyone else had to pay an additional \$1.09 to cover the cost of the cake. How many employees does Harri have at the pet store?

Solution

Solution 1

Let n represent the number of employees that Harri has at the pet store. Then $(n - 3)$ represents the number of employees who actually paid for the cake.

The n employees had each agreed to pay $\frac{\$65.40}{n}$. However, $(n - 3)$ employees actually each paid $\frac{\$65.40}{n-3}$, which was more than the amount they had originally agreed to. The difference between the two amounts is \$1.09. It follows that

$$\begin{aligned} \frac{65.40}{n-3} - \frac{65.40}{n} &= 1.09 \\ \frac{65.40}{n-3}(n)(n-3) - \frac{65.40}{n}(n)(n-3) &= 1.09(n)(n-3) \\ 65.40n - 65.40(n-3) &= 1.09(n)(n-3) \\ 65.40n - 65.40n + 196.2 &= 1.09(n)(n-3) \\ \frac{196.2}{1.09} &= n(n-3) \\ 180 &= n(n-3) \end{aligned}$$

From here we see that we are looking for two positive integers that differ by 3 and multiply to 180. We notice that $15 \times 12 = 180$, and $15 - 12 = 3$. In fact these are the only two positive integers that differ by 3 and multiply to 180. It follows that $n = 15$. Thus, Harri has 15 employees at the pet store.

Solution 2

This solution builds onto Solution 1 by solving the problem algebraically. Note that this level of mathematics is often not taught until grade 10.

Start with Solution 1 and proceed until you reach $180 = n(n - 3)$. From there,

$$\begin{aligned} 180 &= n^2 - 3n \\ 0 &= n^2 - 3n - 180 \\ 0 &= (n - 15)(n + 12) \end{aligned}$$

Therefore, $n = 15$ or $n = -12$. However, since $n > 0$, it follows that $n = 15$. Thus, Harri has 15 employees at the pet store.

**Solution 3**

Let n represent the number of employees that Harri has at the pet store. Then $(n - 3)$ represents the number of employees who actually paid for the cake. The cost of the cake was \$65.40, or 6540 cents. Since 3 of the employees didn't pay, and at least one employee did pay, then we can assume there are at least 4 employees in total.

We check integer values of n , starting with $n = 4$, and determine the difference between the cost per person when there are n people compared to when there are $(n - 3)$ people, until we find a difference of 109 cents. This is summarized in the table below.

Number of Employees (n)	Original Amount, in cents, per Employee ($\frac{6540}{n}$)	Number of Employees who Paid ($n - 3$)	Amount Actually Paid, in cents, per Employee ($\frac{6540}{n-3}$)	Difference in Amounts, in cents ($\frac{6540}{n-3} - \frac{6540}{n}$)
4	$\frac{6540}{4} = 1635$	$4 - 3 = 1$	$\frac{6540}{1} = 6540$	$6540 - 1635 = 4905$
5	$\frac{6540}{5} = 1308$	$5 - 3 = 2$	$\frac{6540}{2} = 3270$	$3270 - 1308 = 1962$
6	$\frac{6540}{6} = 1090$	$6 - 3 = 3$	$\frac{6540}{3} = 2180$	$2180 - 1090 = 1090$
7	$\frac{6540}{7} \approx 934.29$	$7 - 3 = 4$	$\frac{6540}{4} = 1635$	$1635 - 934.29 = 700.71$
8	$\frac{6540}{8} = 817.5$	$8 - 3 = 5$	$\frac{6540}{5} = 1308$	$1308 - 817.5 = 490.5$
9	$\frac{6540}{9} \approx 726.67$	$9 - 3 = 6$	$\frac{6540}{6} = 1090$	$1090 - 726.67 = 363.33$
10	$\frac{6540}{10} = 654$	$10 - 3 = 7$	$\frac{6540}{7} \approx 934.29$	$934.29 - 654 = 280.29$
11	$\frac{6540}{11} \approx 594.55$	$11 - 3 = 8$	$\frac{6540}{8} = 817.5$	$817.5 - 594.55 = 222.95$
12	$\frac{6540}{12} = 545$	$12 - 3 = 9$	$\frac{6540}{9} \approx 726.67$	$726.67 - 545 = 181.67$
13	$\frac{6540}{13} \approx 503.08$	$13 - 3 = 10$	$\frac{6540}{10} = 654$	$654 - 503.08 = 150.92$
14	$\frac{6540}{14} \approx 467.14$	$14 - 3 = 11$	$\frac{6540}{11} \approx 594.55$	$594.55 - 467.14 = 127.14$
15	$\frac{6540}{15} = 436$	$15 - 3 = 12$	$\frac{6540}{12} = 545$	$545 - 436 = 109$

Therefore, when $n = 15$, the difference in amounts is 109 cents, or \$1.09, as desired. Since the difference is decreasing as n is increasing, this is the only possible value of n . Thus, Harri has 15 employees at the pet store.