Problem of the Week
Problem D and Solution
Lunchtime

Problem

Herman provides ketchup, relish, and mustard for the customers at his hot dog stand. During the lunch rush one day, he recorded how many customers had each of these three condiments. He observed the following:

- The total number of customers was 125, and each ordered a single hot dog.

- 82 customers had ketchup, 47 had relish, and 80 had mustard.

- 32 customers had mustard and ketchup, but not relish.

- 5 customers had mustard and relish, but not ketchup.

- The number of customers who had all three of the condiments was double the number of customers who had none of the condiments.

- The number of customers who had exactly two of the condiments was double the number of customers who had all three of the condiments.
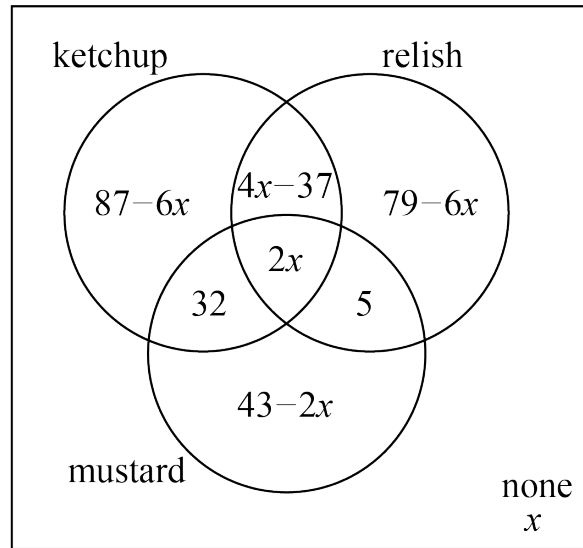
How many customers had ketchup, relish, and mustard on their hot dog?

Solution

Let x be the number of customers who had none of the condiments. Then $2x$ customers had all three of the condiments, and $4x$ customers had exactly two of the condiments.

From the given information, since 32 customers had mustard and ketchup but not relish, and 5 customers had mustard and relish but not ketchup, it follows that $4x - 32 - 5 = 4x - 37$ customers had ketchup and relish, but not mustard.

Also, since 82 customers in total had ketchup, then $82 - 32 - 2x - (4x - 37) = 87 - 6x$ customers had only ketchup. Similarly, since 47 customers in total had relish, then $47 - 5 - 2x - (4x - 37) = 79 - 6x$ customers had only relish. As well, since 80 customers in total had mustard, then $80 - 32 - 2x - 5 = 43 - 2x$ customers had only mustard. We summarize this information in the following Venn diagram.

Since there were 125 customers in total,

$$125 = (87 - 6x) + (4x - 37) + 2x + 32 + (79 - 6x) + 5 + (43 - 2x) + x$$
$$125 = 209 - 7x$$
$$7x = 84$$
$$x = 12$$

Therefore, $2x = 24$ customers had ketchup, relish, and mustard on their hot dog.