

Problem of the Week Problem C and Solution Ones and Zeros

Problem

Determine the three smallest positive integers that contain only 0s and 1s as digits and are divisible by 6.



Solution

A number divisible by 6 must be divisible by both 2 and 3.

Since the number is divisible by 2, it must be even. Since the only digits in our number are 1s and 0s, the units digit must be 0.

If a number is divisible by 3, the sum of the digits must also be divisible by 3. Since the only digits in our number are 1s and 0s, the smallest positive integer divisible by 3 must contain three 1s.

To be as small as possible, the number must contain as few digits as possible. Therefore, the smallest such number contains three 1s and ends in a 0. This number is 1110.

There are no more four-digit positive integers that end in a 0 and contain three 1s. Therefore, the next smallest number will have five digits. To be divisible by 3 and contain five digits, which are only 1s and 0s, the number must contain three 1s and two 0s. As stated above, one 0 must be the units digit. To be as small as possible, the second 0 must be in the highest place value possible. Therefore, the next two smallest positive integers are 10110, and 11010.

Thus, the three smallest positive integers to meet the requirements are 1110, 10110, and 11010.