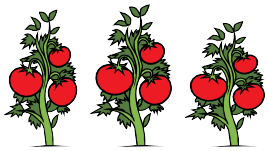

Problem of the Week
Problem C and Solution
Prime Tomatoes

Problem

Antoine has three tomato plants. The total number of tomatoes on all his plants is 41, and one of his plants has exactly 4 more tomatoes than another plant. If the number of tomatoes on each plant is a prime number, determine all possibilities for the number of tomatoes on each plant.

Note: A *prime number* is an integer greater than 1 that has only two positive factors: 1 and itself. The number 17 is prime because its only positive factors are 1 and 17.

Solution

This problem is asking us to find three prime numbers with a sum of 41, where two of the prime numbers differ by exactly 4. We can start by listing all the prime numbers less than 41. They are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37.

Next we can look for all pairs of numbers from this list that differ by 4. These pairs are as follows: 3 and 7, 7 and 11, 13 and 17, and 19 and 23.

Since we want three prime numbers that add to 41, we can look at each of these pairs of numbers to determine all possible solutions.

- 3 and 7: The third number would be $41 - 3 - 7 = 31$. Since this is a prime number, this is a possible solution.
- 7 and 11: The third number would be $41 - 7 - 11 = 23$. Since this is a prime number, this is a possible solution.
- 13 and 17: The third number would be $41 - 13 - 17 = 11$. Since this is a prime number, this is a possible solution.
- 19 and 23: Since $19 + 23 = 42$, this sum is already over 41, so this is not a possible solution.

Therefore, there are three solutions. The tomato plants could have 3, 7, and 31 tomatoes each, 7, 11, and 23 tomatoes each, or 13, 17, and 11 tomatoes each.