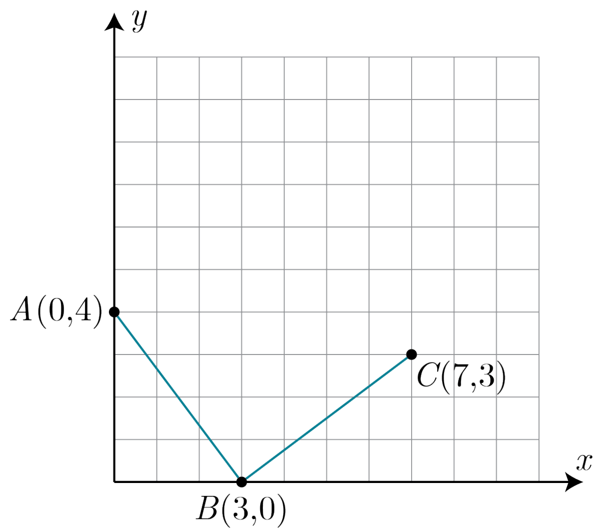
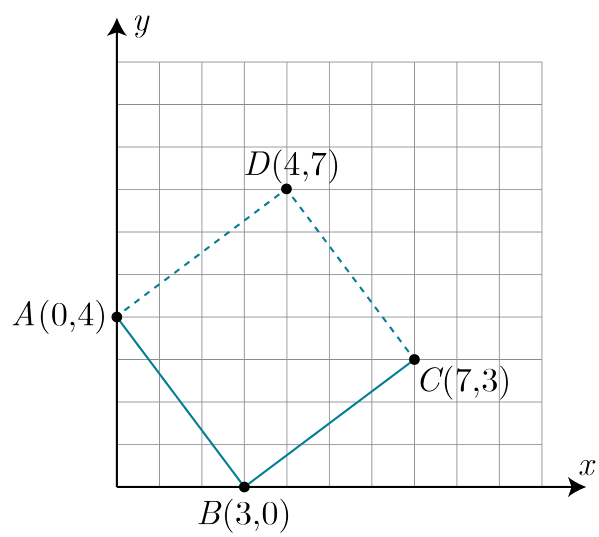
Problem of the Week
Problem C and Solution
Squared Up

Problem
Square $ABCD$ is drawn on graph paper with three of the vertices located at $A(0, 4)$, $B(3, 0)$, and $C(7, 3)$, as shown.



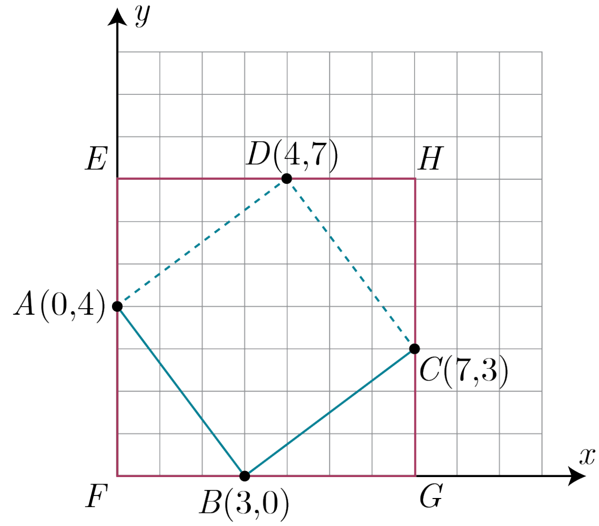
Determine the area of square $ABCD$.

Solution
Solution 1
In this solution we start by determining the coordinates of the fourth vertex. To do this, we observe that to get from A to B we move 3 units right and 4 units down. Then, to get from B to C we move 4 units right and 3 units up. Continuing the pattern, to get from C to D we will move 3 units left and 4 units up to reach $D(4, 7)$. As a check, to get from D to A we move 4 units left and 3 units down, as desired.



Alternatively you could use a protractor to draw sides AD and CD on grid paper, knowing that the corners of a square have right angles. This would also give you $D(4, 7)$.

Next we draw rectangle $EFGH$ with horizontal and vertical sides around $ABCD$ so that each vertex of $ABCD$ lies on one of the sides of the rectangle. This rectangle ends up being a square with side length 7. Inside $EFGH$ is square $ABCD$ as well as four identical right-angled triangles, namely $\triangle AED$, $\triangle AFB$, $\triangle BGC$, and $\triangle CHD$. Each of these right-angled triangles has a base of 4 and a height of 3.



Finally, we calculate the area of $ABCD$. Since the triangles are all identical, their total area is equal to four times the area of any one of them.

$$\begin{aligned}\text{Area } ABCD &= \text{Area } EFGH - 4 \times \text{Area } \triangle AED \\ &= 7 \times 7 - 4 \times (4 \times 3 \div 2) \\ &= 49 - 4 \times 6 \\ &= 49 - 24 = 25\end{aligned}$$

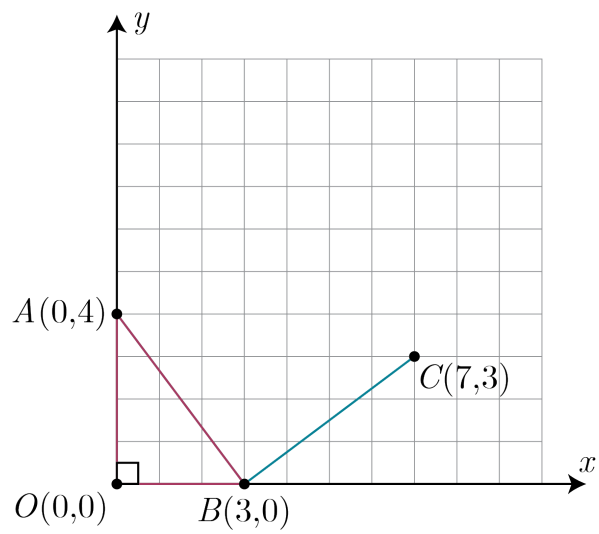
Therefore, the area of $ABCD$ is 25 units^2.

Solution 2

In this solution we will calculate the area of $ABCD$ without determining the coordinates of D. Instead we will use the Pythagorean Theorem.

Since $ABCD$ is a square, in order to calculate its area we need to find the length of only one of its sides. Let $O(0,0)$ be the point at the origin. Then $\triangle AOB$ is a right-angled triangle. The length of side OA is 4 and the length of side OB is 3. We can use the Pythagorean Theorem to calculate AB^2, which is the area of square $ABCD$.

$$\begin{aligned}AB^2 &= OA^2 + OB^2 \\ &= 4^2 + 3^2 \\ &= 16 + 9 = 25\end{aligned}$$



Therefore, the area of $ABCD$ is 25 units^2.