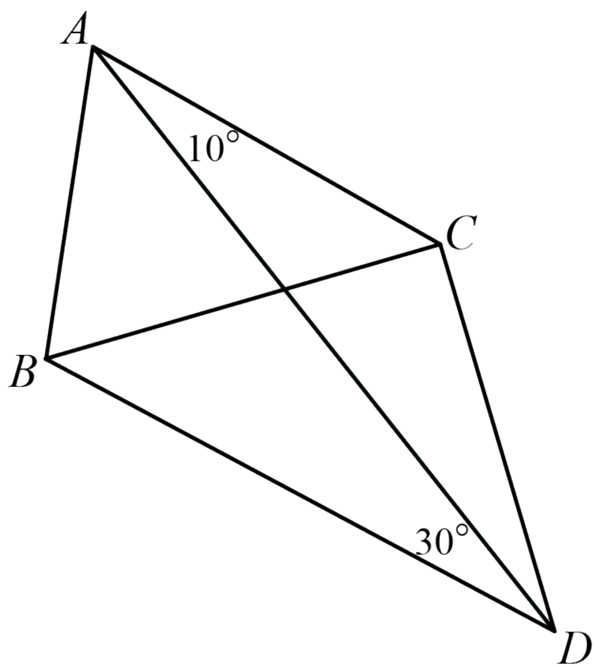
Problem of the Week
Problem C and Solution
Subtle Symmetry

Problem

Quadrilateral $ACDB$ has $AC = BC = DC$, $\angle ADB = 30^\circ$, and $\angle CAD = 10^\circ$.



Determine the measure of $\angle ACB$.

Solution

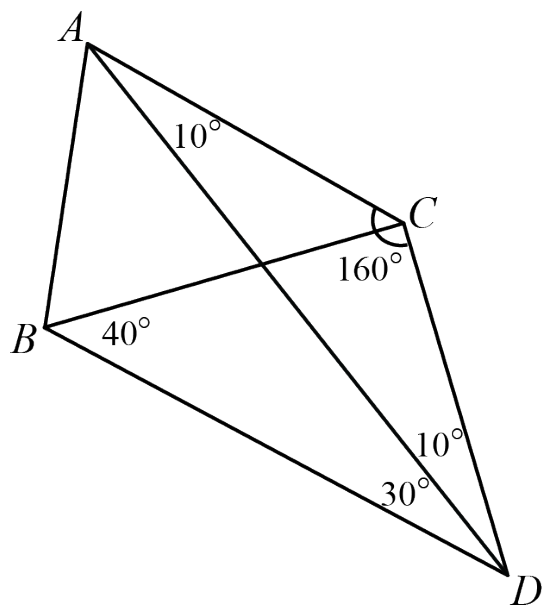
Since $AC = DC$, $\triangle ACD$ is isosceles. Therefore, $\angle CDA = \angle CAD = 10^\circ$.

Thus, $\angle CDB = \angle CDA + \angle ADB = 10^\circ + 30^\circ = 40^\circ$.

Since $BC = DC$, $\triangle BCD$ is isosceles. Therefore, $\angle CBD = \angle CDB = 40^\circ$.

Since the angles in a triangle sum to 180°, in $\triangle ACD$ we have

$$\begin{aligned} \angle CAD + \angle CDA + \angle ACD &= 180^\circ \\ 10^\circ + 10^\circ + \angle ACD &= 180^\circ \\ 20^\circ + \angle ACD &= 180^\circ \\ \angle ACD &= 160^\circ \end{aligned}$$



Since the angles in a triangle sum to 180°, in $\triangle BCD$ we have

$$\begin{aligned}\angle CDB + \angle CBD + \angle BCD &= 180^\circ\\ 40^\circ + 40^\circ + \angle BCD &= 180^\circ\\ 80^\circ + \angle BCD &= 180^\circ\\ \angle BCD &= 100^\circ\end{aligned}$$

Since $\angle ACD = \angle ACB + \angle BCD$, we have $160^\circ = \angle ACB + 100^\circ$. Therefore, $\angle ACB = 160^\circ - 100^\circ = 60^\circ$.

Extension:

Suppose $\angle ADB = 30^\circ$ and $\angle CAD = x^\circ$. Show that $\angle ACB = 60^\circ$.