

Problem of the Week

Problem C and Solution

Tower of Cubes

Problem

Imtiaz is an artist. One of the pieces of art that he created is a tower of three cubes. The bottom cube has a side length of 3 m, the middle cube has a side length of 2 m, and the top cube has a side length of 1 m. The top two cubes are each centred on the cube below.

Imtiaz wishes to paint the piece of art. Since the piece will be suspended in the air, the bottom will also be painted.

Determine the total surface area of the piece of art, including the bottom.

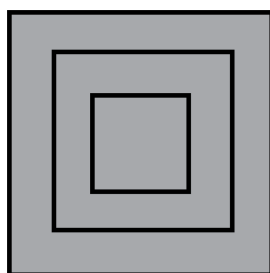
Solution

To determine the areas of the faces, we will use the formula for the area of a rectangle, $\text{Area} = \text{length} \times \text{width}$.

Each cube has 4 exposed square sides, so the total area of all the side faces is

$$\begin{aligned} 4 \times (1 \times 1) + 4 \times (2 \times 2) + 4 \times (3 \times 3) &= 4 \times (1) + 4 \times (4) + 4 \times (9) \\ &= 4 + 16 + 36 \\ &= 56 \text{ m}^2 \end{aligned}$$

To determine the total area of the exposed tops of the cubes, we look down on the tower and see a image like the one below.



This exposed area is equal to the side area of one face of the bottom cube.

Therefore, the total area of the exposed tops of the cubes is $3 \times 3 = 9 \text{ m}^2$. The bottom face to be painted also has area $3 \times 3 = 9 \text{ m}^2$.

Therefore, the total surface area is $56 + 9 + 9 = 74 \text{ m}^2$.

EXTENSION: Three cubes with side lengths x , y and z are stacked on top of each other in a similar manner to the original problem such that $0 < x < y < z$. Show that the total surface area of the tower, including the bottom, is $6z^2 + 4y^2 + 4x^2$.