



Problem of the Week

Problem C and Solution

Three Card Sum

Problem

Five cards are numbered with the integers 1 through 5. The cards are mixed up and placed face down on a table. A person randomly flips over three cards. What is the probability that the sum of the numbers on those three cards is odd?

Solution

In order to determine the probability, we must determine the number of ways to obtain a sum that is odd and divide it by the total number of possibilities for the three flipped cards. We will count the possibilities for the three flipped cards by systematically working through cases.

- Case 1: They flip 1 and 2 and one higher number. Then there are 3 possibilities for the higher number: 3, 4, and 5.
- Case 2: They flip 1 and 3 and one higher number. Then there are 2 possibilities for the higher number: 4 and 5.
- Case 3: They flip 1 and 4 and one higher number. Then there is 1 possibility for the higher number: 5.
- Case 4: They flip 2 and 3 and one higher number. Then there are 2 possibilities for the higher number: 4 and 5.
- Case 5: They flip 2 and 4 and one higher number. Then there is 1 possibility for the higher number: 5.
- Case 6: They flip 3 and 4 and one higher number. Then there is 1 possibility for the higher number: 5.

Therefore, there are $3 + 2 + 1 + 2 + 1 + 1 = 10$ possible flips of three cards. How many of these have an odd sum? We could take each of the possibilities, determine the sum, and then count the number which produce an odd sum. We will present a different method.

The sum of three integers is odd in two instances: all three integers are odd or one integer is odd and the other two are even.

There is only one possibility with three odd integers. This is when 1, 3, and 5 are flipped.

There are three possibilities with one odd integer and two even integers. They are when 1, 2, and 4 are flipped, or when 2, 3, and 4 are flipped, or when 2, 4, and 5 are flipped.

Therefore, there are $1 + 3 = 4$ flips where the sum is odd.

Therefore, the probability of flipping three cards with an odd sum is $\frac{4}{10} = \frac{2}{5}$.

EXTENSION: If nine cards numbered with the integers 1 through 9 are mixed up and placed face down on a table, then the probability of flipping over three cards that have an odd sum is $\frac{10}{21}$. Can you verify this?