



Problem of the Week

Problem C and Solution

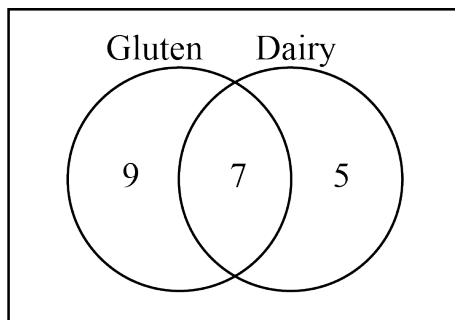
Sweet Treats

Problem

Nadia can confidently make 32 different desserts. Sixteen of the desserts contain gluten and twelve contain dairy. Seven of the desserts contain both gluten and dairy. How many of the desserts are both gluten free and dairy free?

Solution

Since 7 desserts contain both gluten and dairy, and these are included in the 16 desserts that contain gluten, then $16 - 7 = 9$ desserts contain gluten but not dairy. Similarly, the 7 desserts that contain both gluten and dairy are included in the 12 desserts that contain dairy, so $12 - 7 = 5$ desserts contain dairy but not gluten. This is shown in the following Venn diagram.



The desserts can then be separated into four categories: desserts that contain gluten but not dairy, desserts that contain dairy but not gluten, desserts that contain both dairy and gluten, and desserts that are both gluten free and dairy free. If we add the number of desserts in each category, we will get the total number of desserts that Nadia can make, which is 32. Thus, the number of desserts that are both gluten free and dairy free is equal to the total number of desserts minus the number of desserts that contain gluten but not dairy minus the number of desserts that contain dairy but not gluten minus the number of desserts that contain both gluten and dairy.

Since $32 - 9 - 5 - 7 = 11$, it follows that the number of desserts that are both gluten free and dairy free is 11.