Problem of the Week
Problem C and Solution
Bicycle Trip

Problem

Cy Kler has mapped out a 560 km bike route that he wants to complete in seven days. Each day he wants to ride 15 km more than the day before.

If Cy is able to follow his plan, then how many kilometres will he have to ride on the seventh day of the trip?

Solution

Solution 1

We consider how much extra Cy Kler rides on each day after the first.

On Day 2, he rides 15 km extra.
On Day 3, he rides $15 + 15 = 30$ km extra.
On Day 4, he rides $15 + 15 + 15 = 45$ km extra.
On Day 5, he rides $15 + 15 + 15 + 15 = 60$ km extra.
On Day 6, he rides $15 + 15 + 15 + 15 + 15 = 75$ km extra.
On Day 7, he rides $15 + 15 + 15 + 15 + 15 + 15 = 90$ km extra.

Thus, the total distance is made up of seven day trips, each day beginning with the same distance as Day 1 plus $15 + 30 + 45 + 60 + 75 + 90 = 315$ km extra.

So, seven days of riding the same distance as Day 1 would total $560 - 315 = 245$ km. Thus, on Day 1, Cy Kler will ride $245 \div 7 = 35$ km.

On Day 7, Cy Kler will ride the Day 1 distance plus 90 km. Thus, the distance that he will ride on the seventh day is $35 + 90 = 125$ km.

Solutions 2 and 3 present more algebraic approaches to this problem.

Solution 2

Let x be the distance Cy Kler will ride on Day 1. Then he will ride $x + 15$, $x + 30$, $x + 45$, $x + 60$, $x + 75$, and $x + 90$ km on Day 2 through Day 7, respectively. Then,

$$\begin{aligned}
x + (x + 15) + (x + 30) + (x + 45) + (x + 60) + (x + 75) + (x + 90) &= 560 \\
7x + 15 + 30 + 45 + 60 + 75 + 90 &= 560 \\
7x + 315 &= 560 \\
7x + 315 - 315 &= 560 - 315 \\
7x &= 245 \\
\frac{7x}{7} &= \frac{245}{7} \\
x &= 35
\end{aligned}$$

Thus, Cy Kler will ride 35 km on Day 1. Thus, the distance that he will ride on the seventh day is $x + 90 = 35 + 90 = 125$ km.

Solution 3

Let m be the distance Cy will ride on Day 4, the middle day. On Day 5 he would ride $(m + 15)$ km, on Day 6 he would ride $m + 15 + 15 = (m + 30)$ km, and on Day 7 he would ride $m + 30 + 15 = (m + 45)$ km. Working backwards from Day 4, we reduce the distance he rides by 15 km. On Day 3 he would ride $(m - 15)$ km, on Day 2 he would ride $m - 15 - 15 = (m - 30)$ km, and on Day 1 he would ride $m - 30 - 15 = (m - 45)$ km. Then,

$$\begin{aligned}
m + (m + 15) + (m + 30) + (m + 45) + (m - 15) + (m - 30) + (m - 45) &= 560 \\
7m + 15 + 30 + 45 - 15 - 30 - 45 &= 560 \\
7m + 15 - 15 + 30 - 30 + 45 - 45 &= 560 \\
7m &= 560 \\
\frac{7m}{7} &= \frac{560}{7} \\
m &= 80
\end{aligned}$$

Thus, Cy Kler will ride 80 km on Day 4. Thus, the distance that he will ride on the seventh day is $m + 45 = 80 + 45 = 125$ km.

A solution like Solution 3 works well when there are an odd number of terms in a sequence that increases (or decreases) by a constant amount.