



Problem of the Week

Problem C and Solution

Will it be Four or Seven?

Problem

Sophia has been making two types of necklaces: small necklaces that contain four beads each and large necklaces that contain seven beads each.

After creating a certain number of small and large necklaces, a total of 99 beads have been used. Determine all possibilities for how many of each type of necklace Sophia has made.

Solution

Let S represent the number of small necklaces and L represent the number of large necklaces she has made. Since S and L represent numbers of necklaces, both must be integers greater than or equal to 0. Since the small necklaces use 4 beads each, S necklaces would use $4 \times S$ or $4S$ beads in total. Since the large necklaces use 7 beads each, L necklaces would use $7 \times L$ or $7L$ beads in total. Since a total of 99 beads have been used, $4S + 7L = 99$.

We can also determine a maximum value for L . Since each large necklace uses 7 beads, $99 \div 7 \approx 14.1$, and L must be an integer, we know that L must be less than or equal to 14. Thus, L is an integer greater than or equal to 0 and less than or equal to 14. We could at this point check all of the possible integer values for L from 0 to 14. However, we can narrow down the possibilities even more.

In the equation, $4S + 7L = 99$, $4S$ will always be an even integer since 4 times any integer is always even. We have the even integer $4S$ plus $7L$ is equal to the odd integer 99. This means that $7L$ must be an odd integer. (The sum of an even integer and an even integer is an even integer, not an odd integer.) For $7L$ to be an odd integer, L must be odd. (If L is even, $7L$ would be even.) This observation reduces the possible values for L to the odd positive integers between 0 and 14, namely 1, 3, 5, 7, 9, 11, 13. For each possible value of L , we now determine $7L$, the total number of large beads used, $4S = 99 - 7L$, the total number of small beads used, and finally $S = (99 - 7L) \div 4$, the number of small necklaces for that value of L . If S is a non-negative integer, then we have found a valid possibility.

L	$7L$	$4S = 99 - 7L$	$S = (99 - 7L) \div 4$	Valid Possibility?
1	$7 \times 1 = 7$	$99 - 7 = 92$	$92 \div 4 = 23$	Yes, S is an integer
3	$7 \times 3 = 21$	$99 - 21 = 78$	$78 \div 4 = 19.5$	No, S is not an integer
5	$7 \times 5 = 35$	$99 - 35 = 64$	$64 \div 4 = 16$	Yes, S is an integer
7	$7 \times 7 = 49$	$99 - 49 = 50$	$50 \div 4 = 12.5$	No, S is not an integer
9	$7 \times 9 = 63$	$99 - 63 = 36$	$36 \div 4 = 9$	Yes, S is an integer
11	$7 \times 11 = 77$	$99 - 77 = 22$	$22 \div 4 = 5.5$	No, S is not an integer
13	$7 \times 13 = 91$	$99 - 91 = 8$	$8 \div 4 = 2$	Yes, S is an integer

Therefore, Sophia has either made 1 large necklace and 23 small necklaces, or 5 large necklaces and 16 small necklaces, or 9 large necklaces and 9 small necklaces, or 13 large necklaces and 2 small necklaces.