



Problem of the Week

Problem C and Solution

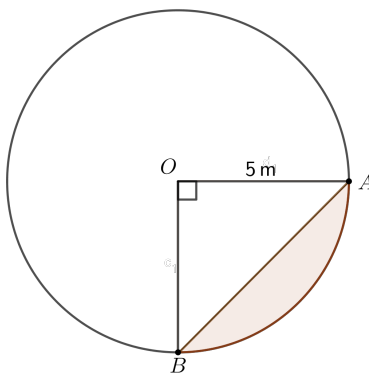
A Wet Sidewalk

Problem

A sprinkler sprays water in a circular region with a radius of 5 m. It is positioned in such a way that some of the water is landing on a nearby sidewalk.

In the diagram, the circle with centre O represents the circular region covered by the water sprinkler. Points A and B lie on the circle with $\angle AOB = 90^\circ$ and $OA = 5$ m. The shaded region, which is the region inside sector AOB but outside of $\triangle AOB$, represents the region of the sidewalk where water is landing.

Determine the area of the region of the sidewalk where the water is landing, correct to one decimal place.



Solution

$\triangle AOB$ along with the shaded region cover $\frac{1}{4}$ of the area of the circle covered by the sprinkler. To determine the area of the shaded region, we need to find the area of the triangle and subtract it from one-quarter of the area of the circle.

Since $\angle AOB = 90^\circ$, we can use OA as the base and OB as the height in the formula for the area of a triangle to find the area of $\triangle AOB$. Further, since OA and OB are radii of the circle, we know that $OA = OB = 5$ m. Therefore,

$$\text{Area } \triangle AOB = \frac{\text{base} \times \text{height}}{2} = \frac{OA \times OB}{2} = \frac{5 \times 5}{2} = \frac{25}{2} = 12.5 \text{ m}^2$$

To determine the area of the quarter circle, we will use the formula for the area of a circle, $A = \pi r^2$, and then divide the result by 4. That is,

$$\text{Area of the Quarter Circle} = \frac{\pi \times r^2}{4} = \frac{\pi \times 5 \times 5}{4} = \frac{\pi \times 25}{4} = (6.25 \times \pi) \text{ m}^2$$

We can now determine the area of the shaded region.

$$\begin{aligned} \text{Area of Shaded Region} &= \text{Area of the Quarter Circle} - \text{Area } \triangle AOB \\ &= 6.25 \times \pi - 12.5 \\ &\approx 7.1 \text{ m}^2 \end{aligned}$$

Therefore, the area of the shaded region, correct to one decimal place, is 7.1 m^2 .