Problème de la Semaine
Problèmes et Solutions 2024-2025
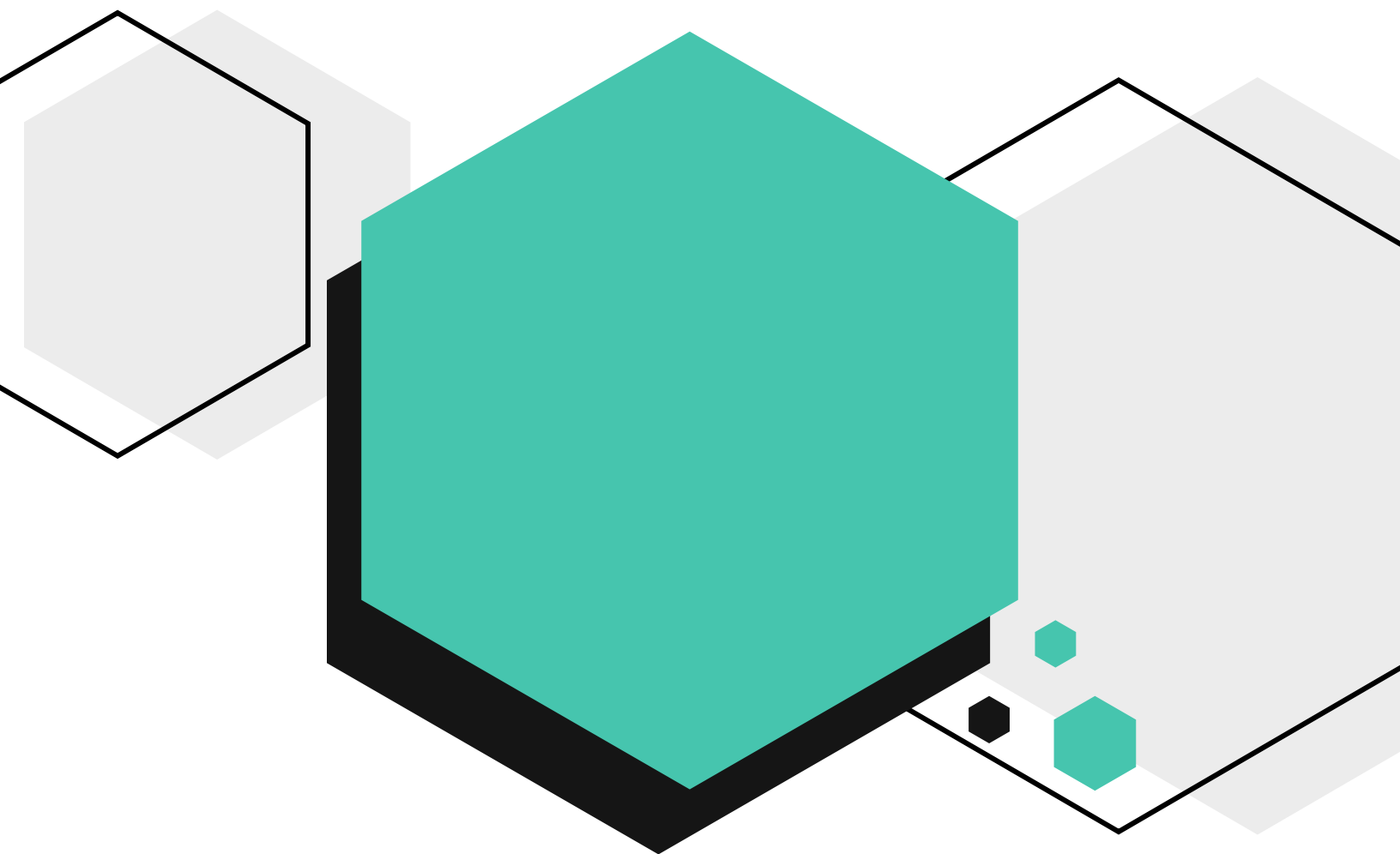
Problème C
7e/8e année

Table des Matières

Les problèmes dans ce livret sont organisés par thème.
Un problème peut apparaître dans plusiers thèmes.
Cliquer sur le nom du thème ci-dessous pour sauter à cette section.

Algèbre (A)

Amène-moi à la couverture
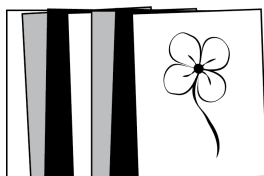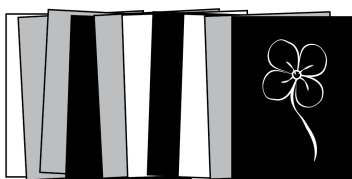
Problème de la semaine
Problème C
Vente de dessins

Éloïse réalise des dessins et les vend dans un parc lors d'un festival d'été. Elle utilise trois couleurs de papier: blanc, gris et noir. Elle dessine d'abord sur du papier blanc et pose le produit final sur la table pour l'exposer. Elle dessine ensuite sur du papier gris, puis sur du papier noir. Chaque fois qu'elle termine un dessin, elle le place sur la pile de dessins qu'elle met en exposition. Elle suit toujours ce même ordre: blanc, gris, noir, si bien qu'après avoir réalisé sept dessins, la pile sur la table se présenterait ainsi:
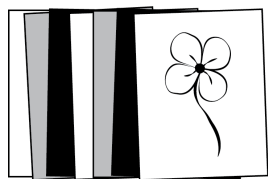

Éloïse commence le festival avec sa pile d'exposition vide et continue de dessiner et de poser ses dessins sur la pile en exposition tout au long du festival. Si quelqu'un veut acheter l'un de ses dessins, il ne peut acheter que le dessin qui se trouve en haut de la pile. À la fin du festival, Éloïse remarque qu'elle a réalisé plus de dessins qu'elle n'en a vendus. Dans la figure ci-dessous, on voit les dessins restants dans la pile.

Quel est le plus petit nombre de dessins qu'Éloïse aurait pu vendre pendant le festival ?

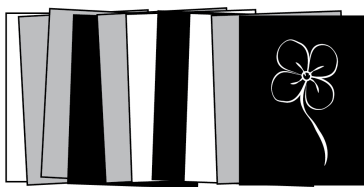Ce problème s'inspire d'un autre problème qui figurait dans un des concours précédents du défi d'informatique Beaver.

THÈMES : ALGÈBRE, PENSÉE COMPUTATIONNELLE

Problem of the Week
Problem C and Solution
Selling Sketches

Problem

Elouise sketches pictures and sells them in the park during a summer festival. She uses three colours of paper: white, grey, and black. First she sketches on white paper and puts it on the table for display. Then she sketches on grey paper, and then she sketches on black paper. After she has finished a sketch, she places it on top of the display pile. She continues in this pattern of white, grey, black, so that after completing seven sketches, her display pile would be as shown above. Elouise starts the festival with nothing in her display pile and continues sketching and adding pictures to the display pile throughout the festival. If someone wants to buy one of her sketches, they can only buy the sketch that is currently on the top of the pile. At the end of the festival, Elouise notes that she has sketched more pictures than she has sold, and the remaining sketches in her display pile are as shown.

What is the fewest number of sketches that Elouise could have sold during the festival?

This problem was inspired by a past Beaver Computing Challenge (BCC) problem.

Solution

Let W represent a sketch on white paper, G represent a sketch on grey paper, and B represent a sketch on black paper. Then the sequence in the display pile at the end of the festival is $WGGBGWBWGB$, where the leftmost sketches represent those on the bottom of the pile.

If no sketches were sold at all, the sequence would be $WGBWGBWGB\ldots$.

We will place a _ in our display pile sequence at the places where we know some sketches must have been sold, because it does not follow the sequence in which they were made. This gives $WG\_GB\_G\_W\_BWGB$. Moving from left to right, the fewest number of sketches that could fill the first blank is 2, namely BW. The fewest number of sketches that could fill the second blank is 1, namely W. The fewest number of sketches that could fill the third blank is 1, namely B. The fewest number of sketches that could fill the last blank is 1, namely G. Filling these in gives $WG\underline{BW}GB\underline{W}G\underline{B}W\underline{G}BWGB$, as desired.

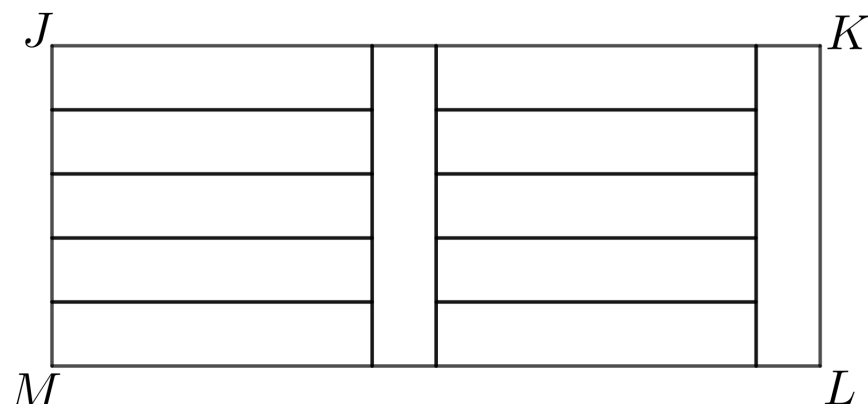Therefore, the minimum number of sketches that Elouise could have sold is $2 + 1 + 1 + 1 = 5$.

Problème de la semaine
Problème C
Un rectangle de rectangles

Dans la figure ci-dessous, le grand rectangle $JKLM$ est formé de douze petits rectangles congrus.

Sachant que $JKLM$ a une aire de 540 cm^2, détermine les dimensions des petits rectangles.

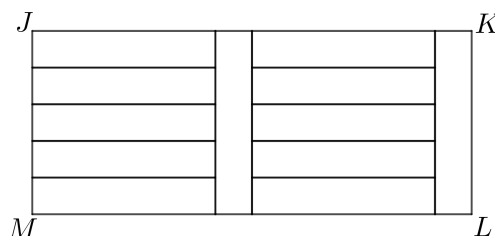**THÈMES :** ALGÈBRE, GÉOMÉTRIE ET MESURE

Problem of the Week
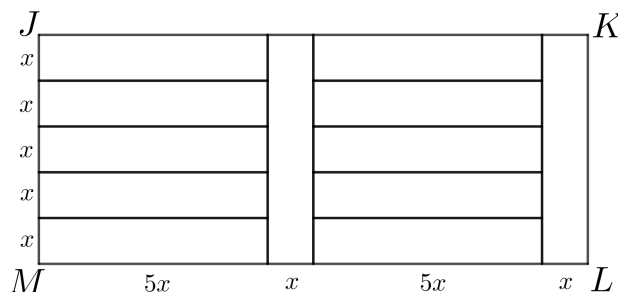## Problem C and Solution
A Rectangle of Rectangles

Problem

Large rectangle $JKLM$ is formed by twelve identical smaller rectangles, as shown.

If the area of $JKLM$ is 540 cm^2, then determine the dimensions of the smaller rectangles.

Solution

Let x be the width of one of the smaller identical rectangles, in cm. Five of the smaller rectangles are stacked on top of each other forming JM, so $JM = x + x + x + x + x = 5x$. Since $JKLM$ is a rectangle, $JM = KL = 5x$. Thus, $5x$ is also the length of a smaller rectangle. Therefore, a smaller rectangle is $5x$ cm by x cm.

From here, we proceed with two different solutions.

Solution 1

Since $JKLM$ is formed by twelve identical smaller rectangles, the area of rectangle $JKLM$ is equal to 12 times the area of one of the smaller rectangles.

$$\text{Area } JKLM = 12 \times \text{Area of one smaller rectangle}$$
$$540 = 12 \times 5x \times x$$
$$540 = 60 \times x^2$$

Dividing both sides by 60, we obtain $x^2 = 9$. Since x is the width of a smaller rectangle, $x > 0$, and so $x = 3$ follows.

Thus, the width of a smaller rectangle is $x = 3$ cm and the length of a smaller rectangle is $5x = 5(3) = 15$ cm.

Therefore, the smaller rectangles are each 15 cm by 3 cm.

Solution 2

Side length ML is made up of the lengths of two of the smaller rectangles plus the widths of two of the smaller rectangles. Therefore, $ML = 5x + 5x + x + x = 12x$ and rectangle $JKLM$ is $12x$ cm by $5x$ cm.

To find the area of $JKLM$ we multiply the length ML by the width JM.

$$\text{Area } JKLM = ML \times JM$$
$$540 = (12x) \times (5x)$$
$$540 = 12 \times 5 \times x \times x$$
$$540 = 60 \times x^2$$

Dividing both sides by 60, we obtain $x^2 = 9$. Since x is the width of a smaller rectangle, $x > 0$, and so $x = 3$ follows.

Thus, the width of a smaller rectangle is $x = 3$ cm and the length of a smaller rectangle is $5x = 5(3) = 15$ cm.

Therefore, the smaller rectangles are each 15 cm by 3 cm.

Problème de la semaine
Problème C
La suite de l'année

Dans une suite de nombres, chaque nombre est appelé un *terme*. Dans la suite 2, 4, 6, 8, le premier terme est 2, le deuxième terme est 4, le troisième terme est 6 et le quatrième terme est 8.

Le premier terme d'une autre suite est 24. On peut déterminer les termes suivants de cette suite de la manière suivante:

— Si un terme est pair, on le divise par 2 pour obtenir le terme suivant.

— Si un terme est impair, on le multiplie par 3, puis on ajoute 1 au produit pour obtenir le terme suivant.

En procédant ainsi, on peut déterminer que les trois premiers termes de la suite sont 24, 12 et 6.

Quel est le 2024e terme de la suite?

THÈMES : ALGÈBRE, SENS DU NOMBRE

Problem of the Week
Problem C and Solution
The Sequence of the Year

Problem

In a sequence of numbers, each number in the sequence is called a *term*. In the sequence 2, 4, 6, 8, the first term is 2, the second term is 4, the third term is 6, and the fourth term is 8.

In another sequence, the first term is 24. We can determine the next terms in the sequence as follows:

- If a term is even, then divide it by 2 to get the next term.

- If a term is odd, then multiply it by 3 and add 1 to get the next term.

By doing this, we can determine that the first three terms in the sequence are 24, 12, and 6. What is the 2024th term in the sequence?

Solution

We will begin by finding more terms in the sequence. The first 14 terms of the sequence are 24, 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1.

If we continue the sequence, we will see that the terms 4, 2, and 1 will continue to repeat. Thus, the 9th term, 12th term, 15th term, and so on, will each have a value of 4. Notice that these term numbers are all multiples of 3. It follows that every term number after 9 that is a multiple of 3 will have a value of 4.

Thus, since 2022 is a multiple of 3, the 2022nd term will have a value of 4. Then, the 2023rd term will have a value of 2, and the 2024th term will have a value of 1.

EXTENSION:

In 1937, the mathematician Lothat Collatz wondered if any sequence whose terms after the first are determined in this way would always eventually reach the number 1, regardless of which number you started with. This problem is actually still unsolved today and is called the Collatz Conjecture.
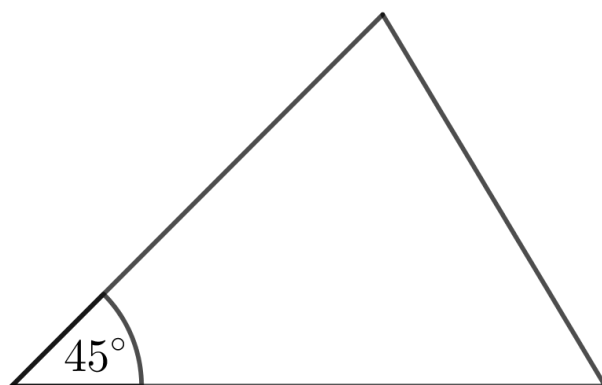
Problème de la semaine
Problème C
Deux angles sur trois

En mesurant les angles d'un triangle, Patricia a trouvé que l'un des angles mesure 45°. Elle a également remarqué que les mesures des deux autres angles du triangle sont dans un rapport de 4 : 5. Quelles sont les mesures de ces deux angles?
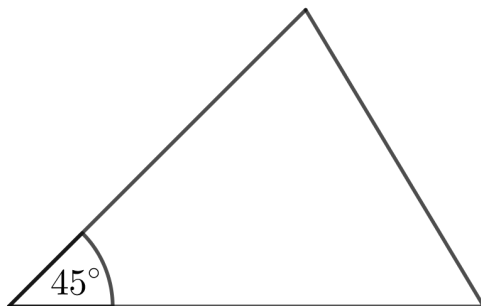

THÈMES : ALGÈBRE, GÉOMÉTRIE ET MESURE, SENS DU NOMBRE

Problem of the Week
Problem C and Solution
Two Out of Three Angles

Problem

While measuring the angles in a triangle, Patricia found the measure of one of the angles is $45°$. Once she had measured the other two angles, she noticed that the measures of these two angles are in the ratio $4 : 5$. What is the measure of each of the other two angles?

Solution

Solution 1

Since the measure of the unknown angles are in the ratio of $4 : 5$, then we can give the angles measures of $5n°$ and $4n°$.

Since the sum of the measures of the three angles in any triangle is $180°$, then

$$45 + 4n + 5n = 180$$
$$4n + 5n = 135$$
$$9n = 135$$
$$n = 15$$

Therefore, the other two angles are $5n = 5(15) = 75°$ and $4n = 4(15) = 60°$.

Solution 2

Since the sum of the measures of the three angles in any triangle is $180°$, then the sum of the measures of the two unknown angles in the triangle is
$180° - 45° = 135°$.

The measures of the two unknown angles are in the ratio $4 : 5$, and so one of the two angles measures $\frac{5}{4+5} = \frac{5}{9}$ of the sum of the two angles, while the other angle measures $\frac{4}{4+5} = \frac{4}{9}$ of the sum of the two angles.

That is, the larger of the two unknown angles measures $\frac{5}{9} \times 135° = 75°$, and the smaller of the unknown angles measures $\frac{4}{9} \times 135° = 60°$.

Therefore, the other two angles are $75°$ and $60°$.

Problème de la semaine
Problème C
C'est le temps de déménager

La famille de Stefania est à la recherche d'un nouvel appartement et Stefania espère qu'ils adopteront un chien après le déménagement. Parmi les 65 appartements qu'elle a trouvés en ligne, 40 sont à proximité d'un parc, 28 autorisent les chiens et 8 ne sont pas à proximité d'un parc et n'autorisent pas les chiens. Combien d'appartements sont à proximité d'un parc et autorisent les chiens ?

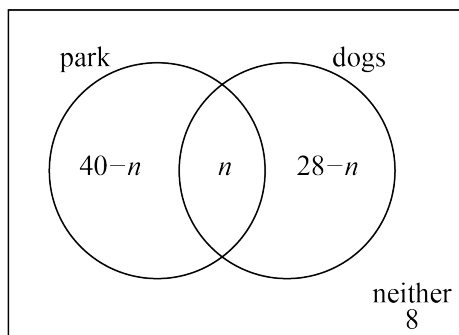# Problem of the Week
Problem C and Solution
Moving Time

Problem

Stefania's family is looking for a new apartment and Stefania is hoping to get a dog after they move. Of the 65 apartments she has found online, 40 are close to a park, 28 allow dogs, and 8 are not close to a park and do not allow dogs. How many of the apartments are close to a park and allow dogs?

Solution

Solution 1

Let n be the number of apartments that are close to a park and allow dogs. Since 40 apartments are close to a park and n apartments are close to a park and allow dogs, then $40 - n$ apartments are close to a park but do not allow dogs. Similarly, since 28 apartments allow dogs and n apartments are close to a park and allow dogs, then $28 - n$ apartments allow dogs but are not close to a park. We also know that 8 apartments are not close to a park and do not allow dogs. This information is summarized in the following Venn diagram.

Since there are 65 apartments in total, then

$$65 = (40 - n) + n + (28 - n) + 8$$
$$65 = 76 - n$$
$$n = 76 - 65 = 11$$

Therefore, 11 of the apartments are close to a park and allow dogs.

Solution 2

Since 8 of the 65 apartments are not close to a park and do not allow dogs, we know that $65 - 8 = 57$ apartments must be close to a park or allow dogs, or both. We know that the 40 apartments close to a park will include the apartments that are close to a park and also allow dogs. Similarly, the 28 apartments that allow dogs will include the apartments that are close to a park and also allow dogs. Thus, if we consider $40 + 28 = 68$, we will be double counting the apartments that are close to a park and also allow dogs. Since there are 57 apartments that are close to a park or allow dogs, or both, we have double-counted $68 - 57 = 11$ apartments. Thus, there must be 11 apartments that are both close to a park and allow dogs.

Problème de la semaine
Problème C
Que des billes !

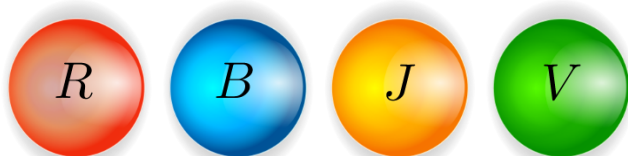Une boîte contient 6 billes rouges, 5 billes bleues, 2 billes jaunes et 3 billes vertes. Plusieurs billes oranges sont ajoutées à la boîte. Toutes les billes de la boîte sont identiques, sauf en couleur.

Une bille est sélectionnée aléatoirement de la boîte et la probabilité que la bille sélectionnée soit bleue ou verte est de $\frac{2}{7}$.

Combien de billes oranges ont été ajoutées à la boîte ?

Problem of the Week
Problem C and Solution
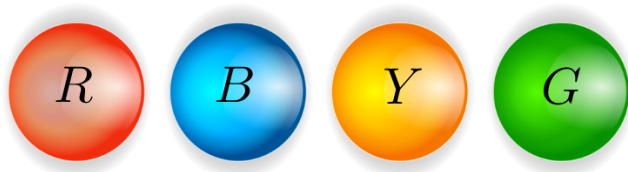## Marbles, Marbles

Problem

A box contains 6 red marbles, 5 blue marbles, 2 yellow marbles, and 3 green marbles. Several orange marbles are added to the box. All the marbles in the box are identical except for colour.

A marble is then randomly selected from the box, and the probability that a blue or green marble is selected is $\frac{2}{7}$.

How many orange marbles were added to the box?

Solution

The number of blue and green marbles in the box is $5 + 3 = 8$.

Let n be the total number of marbles in the box after adding some orange marbles. Since the probability of picking a blue or green marble is $\frac{2}{7}$, we must have $\frac{8}{n} = \frac{2}{7}$.

If we multiply the numerator and denominator of the fraction $\frac{2}{7}$ by 4, we obtain $\frac{2}{7} = \frac{2 \times 4}{7 \times 4} = \frac{8}{28}$. Therefore, $\frac{8}{n} = \frac{8}{28}$. Since the fractions are equal and the numerators are equal, the denominators must also be equal. It follows that $n = 28$.

In the beginning, there were $5 + 6 + 3 + 2 = 16$ marbles in the box. Since there were 16 marbles in the box and there are now 28 marbles in the box, then $28 - 16 = 12$ orange marbles were added to the box.

Therefore, 12 orange marbles were added to the box.

Problème de la semaine
Problème C
Le nombre 1000, c'est toute une somme !

Savais-tu que le nombre 1000 peut s'écrire comme la somme de 16 nombres entiers consécutifs ?

Cela donne,

$$1000 = 55+56+57+58+59+60+61+62+63+64+65+66+67+68+69+70$$

Le *symbole de sommation Sigma*, utilisé dans l'équation ci-dessous, permet d'écrire de façon condensée la somme des nombres démontrée ci-haut.

$$\sum_{i=55}^{70} i = 1000$$

Quel est le nombre minimum de nombres entiers consécutifs que tu peux utiliser dans une équation dont la somme est de 1000 en utilisant au moins deux nombres entiers ?

THÈMES : SENS DU NOMBRE, ALGÈBRE

$$\sum_{i=55}^{70} i = 1000$$

Problem of the Week
Problem C and Solution
1000 is SUM Number!

Problem

Did you know that 1000 can be written as the sum of 16 consecutive integers?

That is,

$$1000 = 55 + 56 + 57 + 58 + 59 + 60 + 61 + 62 + 63 + 64 + 65 + 66 + 67 + 68 + 69 + 70$$

Using at least two integers, what is the minimum number of consecutive integers that sum to exactly 1000?

Solution

We will start with using two integers and then increase the number of integers by 1, until we discover the first number of integers that works.

- Can 1000 be written as the sum of two consecutive integers?

  Let n and $n + 1$ represent the two integers. Then we have

  $$n + (n + 1) = 1000$$
  $$2n + 1 = 1000$$
  $$2n = 999$$
  $$n = 499.5$$

  Since n is not an integer, it is not possible to write 1000 using two consecutive integers.

- Can 1000 be written as the sum of three consecutive integers?

  Let n, $n + 1$, $n + 2$ represent the three integers. Then we have

  $$n + (n + 1) + (n + 2) = 1000$$
  $$3n + 3 = 1000$$
  $$3n = 997$$
  $$n \approx 332.3$$

  Since n is not integer, it is not possible to write 1000 using three consecutive integers. (Refer to the note following the solution for an alternate way to represent three consecutive integers.)

- Can 1000 be written as the sum of four consecutive integers?

  Let n, $n + 1$, $n + 2$, $n + 3$ represent the four integers. Then we have

  $$n + (n + 1) + (n + 2) + (n + 3) = 1000$$
  $$4n + 6 = 1000$$
  $$4n = 994$$
  $$n = 248.5$$

  Since n is not an integer, it is not possible to write 1000 using four consecutive integers.

- Can 1000 be written as the sum of five consecutive integers?

  Let n, $n+1$, $n+2$, $n+3$, $n+4$ represent the five integers. Then we have

$$n + (n+1) + (n+2) + (n+3) + (n+4) = 1000$$
$$5n + 10 = 1000$$
$$5n = 990$$
$$n = 198$$

  Since n is an integer, it is possible to write 1000 using five consecutive integers. That is, as $1000 = 198 + 199 + 200 + 201 + 202$.

Therefore, the minimum number of consecutive integers that sum to 1000 is five.

NOTE:

In the second case, when we checked if 1000 could be written as the sum of three consecutive integers, we could have proceeded as follows:

Let $a - 1$, a, $a + 1$ represent the three consecutive integers. Then we have

$$(a - 1) + a + (a + 1) = 1000$$
$$3a = 1000$$
$$a \approx 333.3$$

Since a is not an integer, it is not possible to write 1000 using three consecutive integers.

This idea is useful when we are finding the sum of an odd number of consecutive integers. If we applied the same idea to the fourth case by using $a - 2$, $a - 1$, a, $a + 1$, $a + 2$ to represent the five consecutive integers, we would have

$$(a - 2) + (a - 1) + a + (a + 1) + (a + 2) = 1000$$
$$5a = 1000$$
$$a = 200$$

Thus, $a - 2 = 198$, $a - 1 = 199$, $a + 1 = 201$, and $a + 2 = 202$. And so it is possible to write 1000 using five consecutive integers as $1000 = 198 + 199 + 200 + 201 + 202$.

FOR FURTHER THOUGHT:

What is the largest **odd** number of consecutive positive integers that can be used to sum to 1000?

How would your answer change if the word positive was removed from the above sentence?
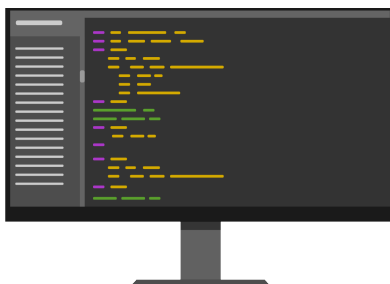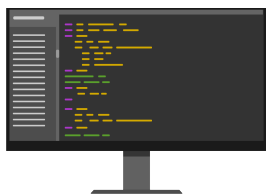
Problème de la semaine
Problème C
Programme de sommes

Ruben a écrit un programme qui utilise une liste de nombres comme données d'entrée. Le programme additionne ensuite chaque paire de nombres adjacents de la liste pour obtenir une nouvelle liste. Ensuite, il répète ce processus avec la nouvelle liste jusqu'à ce qu'il ne reste plus qu'un seul nombre. Ce nombre constitue alors la sortie.

Par exemple, si l'entrée est $(1,\ 5,\ 3,\ 2)$, le programme additionne les nombres adjacents pour obtenir $(6,\ 8,\ 5)$. Il répète ensuite le processus pour obtenir $(14,\ 13)$, puis (27). Comme la liste ne contient plus qu'un seul nombre, la sortie est 27.

Soit R le nombre préféré de Ruben. Ruben a donc entré les nombres $(6,\ 4,\ R,\ 7)$ dans son programme et la sortie était 4 fois son nombre préféré, soit 4R. Quel est alors le nombre préféré de Ruben ?

THÈMES : Algèbre, Sens du nombre

Problem of the Week
Problem C and Solution
Sum Program

Problem

Ruben wrote a program that takes a list of numbers as input. The program then adds each pair of adjacent numbers in the list to obtain a new list of numbers, then repeats this process with the new list until the list contains only one number. This number is then the output.

For example, if the input is $(1, 5, 3, 2)$ then the program adds the adjacent numbers in the list to obtain $(6, 8, 5)$. The program then repeats this process to obtain $(14, 13)$, and then (27). Since the list now contains only one number, the output is 27.

Let R represent Ruben's favourite number. Ruben input the numbers $(6, 4, R, 7)$ into his program and the output was 4 times his favourite number, or $4R$. What is Ruben's favourite number?

Solution

We could attempt this problem using trial and error, and perhaps we would even stumble on the correct answer. However, this may take a long time and is not an efficient method for solving the problem. Instead we will provide a more algebraic solution.

The list of numbers inputted into the program was $(6, 4, R, 7)$. After the adjacent numbers are added together, we obtain $(10, 4 + R, R + 7)$. After the adjacent numbers in this list are added together, we obtain $(10 + 4 + R, 4 + R + R + 7)$, which is equal to $(14 + R, 11 + 2R)$. After the adjacent numbers in this list are added together, we obtain $(14 + R + 11 + 2R)$ which is equal to $(25 + 3R)$. Since there is now one number in the list, this number should be equal to the output of the program, which is $4R$. Thus,

$$25 + 3R = 4R$$
$$25 + 3R - 3R = 4R - 3R$$
$$25 = R$$

Therefore, Ruben's favourite number is 25.
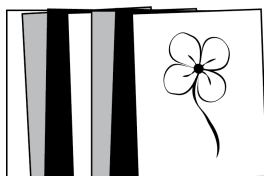
Raisonnement informatiques (C)
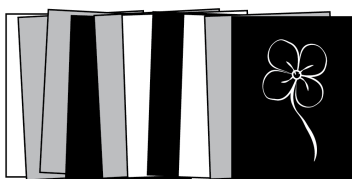
Amène-moi à la couverture

Problème de la semaine
Problème C
Vente de dessins

Éloïse réalise des dessins et les vend dans un parc lors d'un festival d'été. Elle utilise trois couleurs de papier: blanc, gris et noir. Elle dessine d'abord sur du papier blanc et pose le produit final sur la table pour l'exposer. Elle dessine ensuite sur du papier gris, puis sur du papier noir. Chaque fois qu'elle termine un dessin, elle le place sur la pile de dessins qu'elle met en exposition. Elle suit toujours ce même ordre: blanc, gris, noir, si bien qu'après avoir réalisé sept dessins, la pile sur la table se présenterait ainsi:

Éloïse commence le festival avec sa pile d'exposition vide et continue de dessiner et de poser ses dessins sur la pile en exposition tout au long du festival. Si quelqu'un veut acheter l'un de ses dessins, il ne peut acheter que le dessin qui se trouve en haut de la pile. À la fin du festival, Éloïse remarque qu'elle a réalisé plus de dessins qu'elle n'en a vendus. Dans la figure ci-dessous, on voit les dessins restants dans la pile.

Quel est le plus petit nombre de dessins qu'Éloïse aurait pu vendre pendant le festival ?

Ce problème s'inspire d'un autre problème qui figurait dans un des concours précédents du défi d'informatique Beaver.

Thèmes : Algèbre, Pensée computationnelle

Problem of the Week
Problem C and Solution
Selling Sketches

Problem

Elouise sketches pictures and sells them in the park during a summer festival. She uses three colours of paper: white, grey, and black. First she sketches on white paper and puts it on the table for display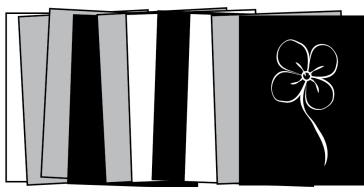. Then she sketches on grey paper, and then she sketches on black paper. After she has finished a sketch, she places it on top of the display pile. She continues in this pattern of white, grey, black, so that after completing seven sketches, her display pile would be as shown above. Elouise starts the festival with nothing in her display pile and continues sketching and adding pictures to the display pile throughout the festival. If someone wants to buy one of her sketches, they can only buy the sketch that is currently on the top of the pile. At the end of the festival, Elouise notes that she has sketched more pictures than she has sold, and the remaining sketches in her display pile are as shown.

What is the fewest number of sketches that Elouise could have sold during the festival?

This problem was inspired by a past Beaver Computing Challenge (BCC) problem.

Solution

Let W represent a sketch on white paper, G represent a sketch on grey paper, and B represent a sketch on black paper. Then the sequence in the display pile at the end of the festival is $WGGBGWBWGB$, where the leftmost sketches represent those on the bottom of the pile.

If no sketches were sold at all, the sequence would be $WGBWGBWGB\dots$.

We will place a _ in our display pile sequence at the places where we know some sketches must have been sold, because it does not follow the sequence in which they were made. This gives $WG\_GB\_G\_W\_BWGB$. Moving from left to right, the fewest number of sketches that could fill the first blank is 2, namely BW. The fewest number of sketches that could fill the second blank is 1, namely W. The fewest number of sketches that could fill the third blank is 1, namely B. The fewest number of sketches that could fill the last blank is 1, namely G. Filling these in gives $WG\underline{BW}GB\underline{W}G\underline{B}W\underline{G}BWGB$, as desired.

Therefore, the minimum number of sketches that Elouise could have sold is
$2 + 1 + 1 + 1 = 5$.

Problème de la semaine
Problème C
Cinq dans le train ION

Samedi après-midi, Ajay, Badr, Ciro, Dena et Ema sont montés à bord du même train léger sur rail ION à la gare de l'Université de Waterloo. Chaque personne écoutait quelque chose de différent dans son casque d'écoute et chaque personne est descendue à une gare différente. Détermine à quelle gare chaque personne est descendue et ce qu'elle écoutait.

(1) Ciro est descendu du train à la gare Fairway et n'était *pas* la personne qui écoutait les actualités.

(2) La personne qui est descendue à la gare Centrale écoutait un livre audio et n'était *pas* Dena.

(3) La personne qui écoutait une partie de hockey n'était *pas* la personne qui est descendue à la station Borden.

(4) Ema écoutait de la musique et est descendue à la gare Queen.

(5) Ajay écoutait un balado et n'était *pas* la personne qui est descendue à la gare Willis Way.

Problem of the Week
Problem C and Solution
Five on the ION

Problem

On Saturday afternoon, Ajay, Badr, Ciro, Dena, and Ema got on the same ION light rail train at University of Waterloo station. Each person was listening to something different on their headphones and each person got off at a different station. Using the clues below, determine which station each person got off at, and what they were listening to.

(1) Ciro got off the train at Fairway station but was *not* the person who was listening to the news.

(2) The person who got off at Central station was listening to an audiobook, and was *not* Dena.

(3) The person who was listening to a hockey game was *not* the person who got off at Borden station.

(4) Ema was listening to music and got off at Queen station.

(5) Ajay was listening to a podcast but was *not* the person who got off at Willis Way station.

Solution

We can use a table to summarize the information in the clues. We will start by filling in the information from clues (1), (4), and (5). If we know that something does not go in a particular cell, we will also include that in the table.

Person	Station	Listening to
Ajay	*not* Willis Way	podcast
Badr		
Ciro	Fairway	*not* news
Dena		
Ema	Queen	music

From clue (2), we know the person who got off at Central station was listening to an audiobook. The only people who have not been matched with a station or what they were listening to are Badr and Dena. Since clue (2) also tells us this person was not Dena, it follows that Badr got off at Central station and was listening to an audiobook.

Then, since we know someone got off at Willis Way station and it was not Ajay, it follows that it must have been Dena since she's the only remaining person who hasn't been matched with a station.

Similarly, we know someone was listening to the news and it was not Ciro. Thus, it must have been Dena since she's the only remaining person who hasn't been matched with what they were listening to. These updates are shown in the table below.

Person	Station	Listening to
Ajay	*not* Willis Way	podcast
Badr	Central	audiobook
Ciro	Fairway	*not* news
Dena	Willis Way	news
Ema	Queen	music

Finally, clue (3) tells us someone was listening to a hockey game and someone got off at Borden station. Since Ciro is the only person who hasn't been matched with what they're listening to, Ciro must have been listening to a hockey game. Similarly since Ajay is the only person who hasn't been matched with a station, Ajay must have gotten off at Borden station. Thus, our final solution is shown.

Person	Station	Listening to
Ajay	Borden	podcast
Badr	Central	audiobook
Ciro	Fairway	hockey game
Dena	Willis Way	news
Ema	Queen	music

Problème de la semaine
Problème C
Partage équitable

Piero a 7 tuiles, chacune portant un nombre entier différent de 1 à 7. Il existe plusieurs façons de regrouper ses tuiles en formant des ensembles contenant chacun au moins une tuile. Par exemple, il peut diviser ses tuiles en 3 groupes, comme illustré.

$$\boxed{1}\boxed{2}\boxed{4}\boxed{6} \qquad \boxed{3}\boxed{7} \qquad \boxed{5}$$

La somme des nombres dans chacun de ces groupes est respectivement 13, 10 et 5.

Piero décide ensuite de former différents groupes avec ses tuiles et remarque que, cette fois, la somme des nombres dans chaque groupe est identique. De combien de façons différentes peut-il répartir ses tuiles en au moins deux groupes, de sorte que la somme des nombres dans chaque groupe soit la même ?

Problem of the Week
Problem C and Solution
Group Work

Problem

Piero has 7 tiles, each with a different integer from 1 to 7 written on it. There are many ways in which he can separate his tiles into groups, where each group contains at least one tile. For example, he can separate his tiles into 3 groups as shown.

$$\boxed{1}\boxed{2}\boxed{4}\boxed{6} \qquad\qquad \boxed{3}\boxed{7} \qquad\qquad \boxed{5}$$

The sum of the numbers in each of these groups is 13, 10, and 5, respectively.

Piero then separates his tiles into different groups and notices that the sum of the numbers in each group is the same. In how many different ways can Piero separate his tiles into at least two groups so that the sum of the numbers in each group is the same?

Solution

The sum of the numbers on Piero's tiles is $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. If the sum of the numbers in each group is the same, then this sum must be a factor of 28. The positive factors of 28 are 1, 2, 4, 7, 14, and 28. Since the number 7 must be in one of the groups, it follows that the sum of each group must be at least 7. Also since there must be at least 2 groups, then the sum of each group cannot be 28, because in that case there would be only 1 group. It follows that the groups could each have a sum of 7 or a sum of 14. We will look at these two cases.

Case 1: Each group has a sum of 7.
In this case, there would be $28 \div 7 = 4$ groups. The only way to separate the tiles into 4 groups, each with a sum of 7, is as $\{7\}$, $\{1, 6\}$, $\{2, 5\}$, $\{3, 4\}$.

Case 2: Each group has a sum of 14.
In this case, there would be $28 \div 14 = 2$ groups. To help count the possible groups, we note that one of the two groups must contain the tile with the 7. The possibilities for the other tiles are as follows:

- If the group with the 7 also contains the 6, then the only possibility is $\{1, 6, 7\}$. The other group must then be $\{2, 3, 4, 5\}$.

- If the group with the 7 also contains the 5, then the only possibility is $\{2, 5, 7\}$. The other group must then be $\{1, 3, 4, 6\}$.

- If the group with the 7 also contains the 4, then the group could be either $\{3, 4, 7\}$ or $\{1, 2, 4, 7\}$. The other group would then be $\{1, 2, 5, 6\}$ or $\{3, 5, 6\}$, respectively.

- If the group with the 7 also contains the 3, 2, or 1, then it must also contain the 4, 5, or 6, since $7 + 3 + 2 + 1 = 13$, which is less than 14. So there are no additional cases.

Thus, there are 4 ways to separate the tiles into 2 groups, each with a sum of 14.

Therefore, there are $1 + 4 = 5$ different ways in which Piero can separate his tiles into at least two groups so that the sum of the numbers in each group is the same. These are listed below.

- $\{7\}$, $\{1, 6\}$, $\{2, 5\}$, $\{3, 4\}$
- $\{1, 6, 7\}$, $\{2, 3, 4, 5\}$
- $\{2, 5, 7\}$, $\{1, 3, 4, 6\}$
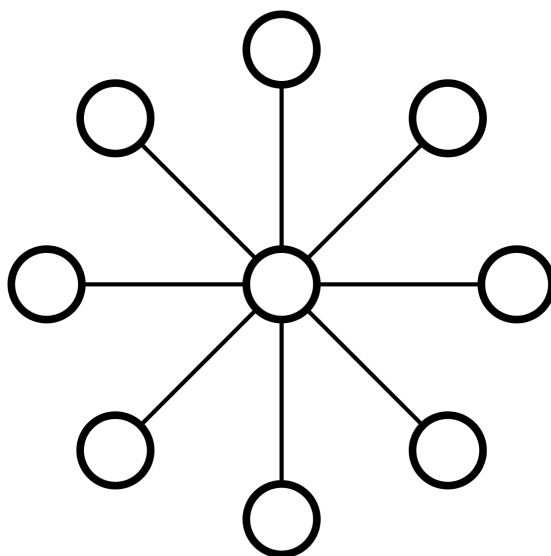- $\{3, 4, 7\}$, $\{1, 2, 5, 6\}$
- $\{1, 2, 4, 7\}$, $\{3, 5, 6\}$

Problème de la semaine
Problème C
Le jeu du flocon

Le jeu du flocon se joue sur un plateau de jeu constitué de 9 cercles. Il y a 1 cercle au centre du plateau, avec 4 segments de droite qui passent par ce cercle, chacun ayant un cercle à chaque extrémité.

Deux joueurs jouent à tour de rôle en plaçant des disques numérotés de 1 à 9 dans les cercles du plateau. Chaque numéro ne peut être utilisé qu'une seule fois pendant la partie. L'objectif du jeu est d'être le premier joueur à placer un disque de manière à ce que la somme des trois nombres alignés le long d'un segment de droite passant par le cercle central soit égale à 15.

Alex et Blake jouent au jeu. Alex commence la partie. Démontre qu'Alex dispose d'une *stratégie gagnante* si elle place un 6 dans le cercle central lors de son premier tour. C'est-à-dire, démontre que si Alex place un 6 dans le cercle central dès son premier coup, elle gagnera toujours, peu importe les nombres joués par Blake.
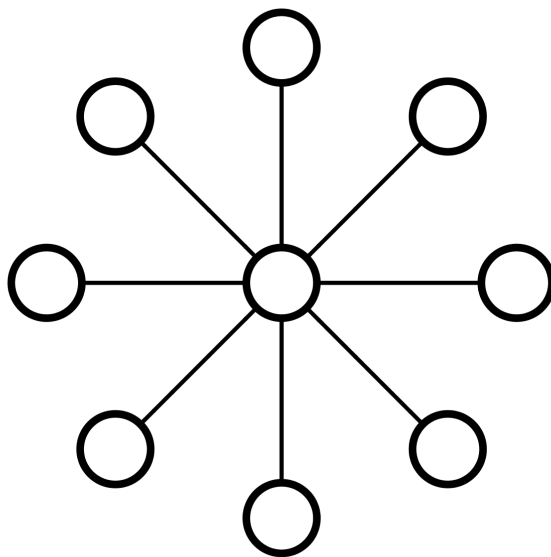
Thème : Pensée computationnelle

Problem of the Week
Problem C and Solution
The Snowflake Game

Problem

The Snowflake Game is played on a board that consists 9 circles. There is 1 circle in the centre of the board, with 4 line segments through it, and a circle at the end of each line segment.

Two players alternate turns placing discs numbered 1 to 9 in the circles on the board. Each number can only be used once in any game. The object of the game is to be the first player to place a disc so that the sum of the three numbers along a line segment through the centre circle is exactly 15.

Alex and Blake play the game. Alex goes first. Show that Alex has a *winning strategy* if she places a 6 in the centre circle on her first turn. That is, show that if Alex places a 6 in the centre circle on her first turn, then no matter which numbers Blake plays, Alex can always win the game.

Solution

If Alex places a 6 in the centre circle on her first turn, then the other two discs in the line segment would need to add to 9 in order for the sum to be 15.

Since each number can be used only once in a game, after Alex places the 6, the numbers that remain are 1, 2, 3, 4, 5, 7, 8, and 9.

- **Case 1:** Blake places a 1, 2, 4, 5, 7, or 8 on his first turn.

  Then there is always an unused number remaining that Alex can place on the same line segment to win the game on her second turn. That is, if Blake

places a 1, then Alex should place an 8. If Blake places a 2, then Alex should place a 7. If Blake places a 4, then Alex should place a 5. If Blake places a 5, then Alex should place a 4. If Blake places a 7, then Alex should place a 2. If Blake places an 8, then Alex should place a 1.

- **Case 2:** Blake places a 3 on his first turn.

  Then the sum of the two discs on the line segment containing the 3 will be 9. Alex cannot win on her second turn since the only way to make the sum in that line segment 15 would be for her to place another 6. No number may be used more than once so this is not possible. However, if Alex completes the line segment by placing a 9 on her second turn, then the remaining discs will have numbers 1, 2, 4, 5, 7, and 8. Then, as in Case 1, no matter what Blake places on his second turn, there will always be a number that Alex can place on that same line segment so that the three numbers on the line segment sum to 15.

- **Case 3:** Blake places a 9 on his first turn.

  Then the sum of the two discs on the line segment containing the 9 will be 15. Alex cannot win on her second turn since playing any other disc on that line segment will make the sum greater than 15. However, if Alex completes the line segment by placing a 3 on her second turn, then the remaining discs will have numbers 1, 2, 4, 5, 7, and 8. Then, as in Case 1, no matter what Blake places on his second turn, there will always be a number that Alex can place on that same line segment so that the three numbers on the line segment sum to 15.

Therefore, we have shown that if Alex places a 6 in the centre circle on her first turn, then no matter which numbers Blake plays, Alex can always win the game on either her second or third turn.

For Further Thought:

What other numbers can Alex place in the middle circle on her first turn to have a winning strategy? That is, what other numbers can Alex place in the middle circle on her first turn so that she can always win the game, no matter which numbers Blake plays?

Gestion des données (D)

Amène-moi à la couverture

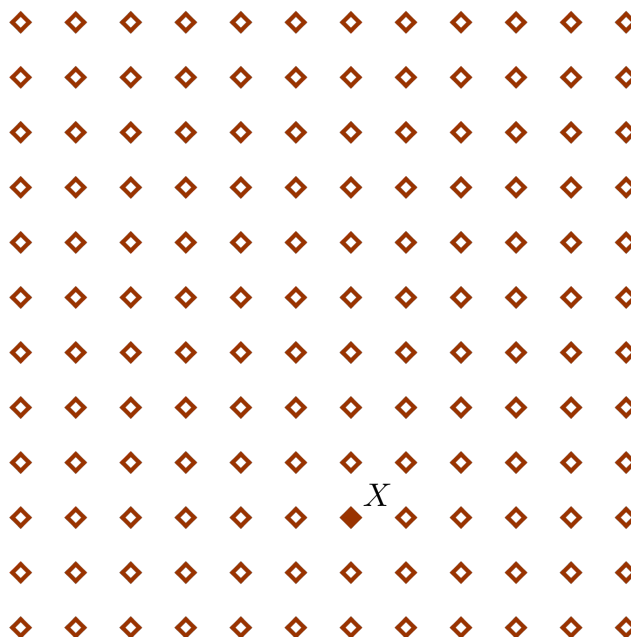# Problème de la semaine
Problème C
Histoire de carreaux

144 carreaux ont été utilisés pour créer la grille ci-dessous de 12 par 12 carreaux. Un de ces carreaux est coloré et étiqueté X.

Un des 143 autres carreaux dans la grille est choisi aléatoirement, puis est coloré et étiqueté Y. Quelle est la probabilité que le segment de droite reliant X et Y soit vertical ou horizontal?

THÈME : GESTION DES DONNÉES

Problem of the Week
Problem C and Solution
Diamond in the Rough

Problem

Using 144 diamonds, the 12 by 12 grid of diamonds below is created. One of the diamonds is coloured and labelled X.

One of the other 143 diamonds in the grid is randomly chosen and is coloured in and labelled Y. What is the probability the line segment connecting X and Y is vertical or horizontal?

Solution

Line segment XY is vertical if Y is chosen from the diamonds in the column in which X lies. In this column there are 11 diamonds other than X which could be chosen to be Y so that XY is vertical.

Line segment XY is horizontal if Y is chosen from the diamonds in the row in which X lies. In this row there are 11 diamonds other than X which could be chosen to be Y so that XY is horizontal. Each of these 11 diamonds is different from the 11 diamonds in the column containing X. Thus, there are $11 + 11 = 22$ diamonds which may be chosen for Y so that XY is vertical or horizontal.

Since there are a total of 143 diamonds to choose Y from, the probability that Y is chosen so that XY is vertical or horizontal is $\frac{22}{143}$ or $\frac{2}{13}$.

Problème de la semaine
Problème C
C'est le temps de déménager

La famille de Stefania est à la recherche d'un nouvel appartement et Stefania espère qu'ils adopteront un chien après le déménagement. Parmi les 65 appartements qu'elle a trouvés en ligne, 40 sont à proximité d'un parc, 28 autorisent les chiens et 8 ne sont pas à proximité d'un parc et n'autorisent pas les chiens. Combien d'appartements sont à proximité d'un parc et autorisent les chiens ?

THÈMES : ALGÈBRE, GESTION DES DONNÉES

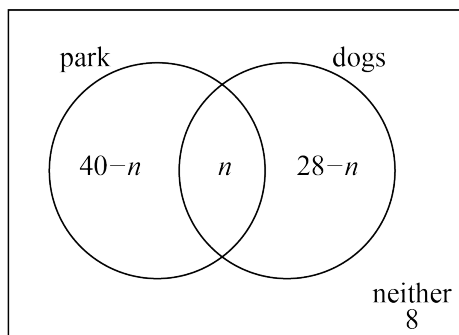# Problem of the Week
Problem C and Solution
Moving Time

Problem

Stefania's family is looking for a new apartment and Stefania is hoping to get a dog after they move. Of the 65 apartments she has found online, 40 are close to a park, 28 allow dogs, and 8 are not close to a park and do not allow dogs. How many of the apartments are close to a park and allow dogs?

Solution

Solution 1

Let n be the number of apartments that are close to a park and allow dogs. Since 40 apartments are close to a park and n apartments are close to a park and allow dogs, then $40 - n$ apartments are close to a park but do not allow dogs. Similarly, since 28 apartments allow dogs and n apartments are close to a park and allow dogs, then $28 - n$ apartments allow dogs but are not close to a park. We also know that 8 apartments are not close to a park and do not allow dogs. This information is summarized in the following Venn diagram.

Since there are 65 apartments in total, then

$$65 = (40 - n) + n + (28 - n) + 8$$
$$65 = 76 - n$$
$$n = 76 - 65 = 11$$

Therefore, 11 of the apartments are close to a park and allow dogs.

Solution 2

Since 8 of the 65 apartments are not close to a park and do not allow dogs, we know that $65 - 8 = 57$ apartments must be close to a park or allow dogs, or both. We know that the 40 apartments close to a park will include the apartments that are close to a park and also allow dogs. Similarly, the 28 apartments that allow dogs will include the apartments that are close to a park and also allow dogs. Thus, if we consider $40 + 28 = 68$, we will be double counting the apartments that are close to a park and also allow dogs. Since there are 57 apartments that are close to a park or allow dogs, or both, we have double-counted $68 - 57 = 11$ apartments. Thus, there must be 11 apartments that are both close to a park and allow dogs.
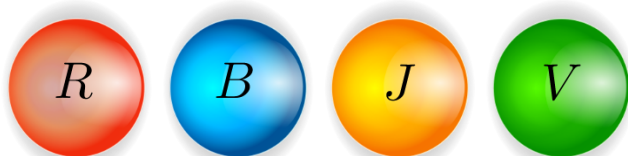
Problème de la semaine
Problème C
Que des billes !

Une boîte contient 6 billes rouges, 5 billes bleues, 2 billes jaunes et 3 billes vertes. Plusieurs billes oranges sont ajoutées à la boîte. Toutes les billes de la boîte sont identiques, sauf en couleur.

Une bille est sélectionnée aléatoirement de la boîte et la probabilité que la bille sélectionnée soit bleue ou verte est de $\frac{2}{7}$.

Combien de billes oranges ont été ajoutées à la boîte ?
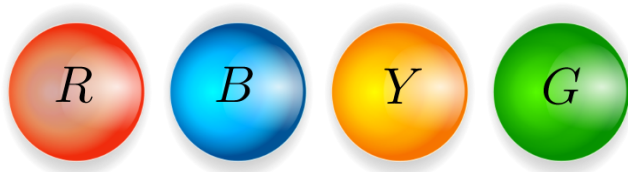
Problem of the Week
Problem C and Solution
Marbles, Marbles

Problem

A box contains 6 red marbles, 5 blue marbles, 2 yellow marbles, and 3 green marbles. Several orange marbles are added to the box. All the marbles in the box are identical except for colour.

A marble is then randomly selected from the box, and the probability that a blue or green marble is selected is $\frac{2}{7}$.

How many orange marbles were added to the box?

Solution

The number of blue and green marbles in the box is $5 + 3 = 8$.

Let n be the total number of marbles in the box after adding some orange marbles. Since the probability of picking a blue or green marble is $\frac{2}{7}$, we must have $\frac{8}{n} = \frac{2}{7}$.

If we multiply the numerator and denominator of the fraction $\frac{2}{7}$ by 4, we obtain $\frac{2}{7} = \frac{2 \times 4}{7 \times 4} = \frac{8}{28}$. Therefore, $\frac{8}{n} = \frac{8}{28}$. Since the fractions are equal and the numerators are equal, the denominators must also be equal. It follows that $n = 28$.

In the beginning, there were $5 + 6 + 3 + 2 = 16$ marbles in the box. Since there were 16 marbles in the box and there are now 28 marbles in the box, then $28 - 16 = 12$ orange marbles were added to the box.

Therefore, 12 orange marbles were added to the box.
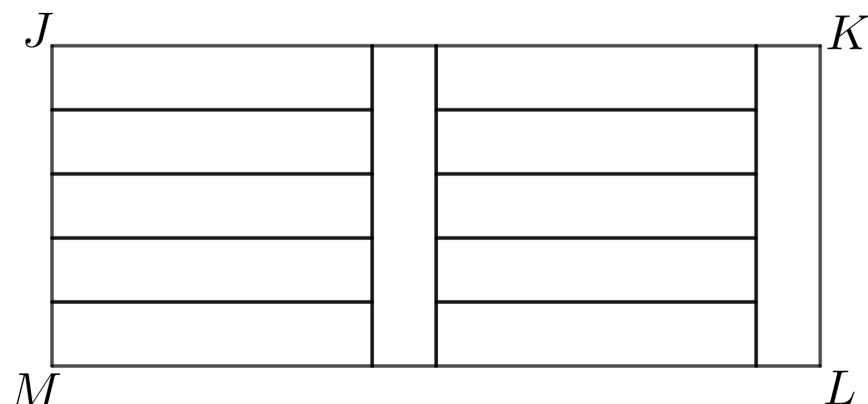
Géométrie et mesure (G)

Amène-moi à la couverture

Problème de la semaine
Problème C
Un rectangle de rectangles

Dans la figure ci-dessous, le grand rectangle $JKLM$ est formé de douze petits rectangles congrus.

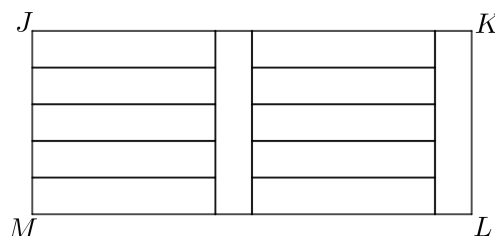Sachant que $JKLM$ a une aire de 540 cm², détermine les dimensions des petits rectangles.

Problem of the Week
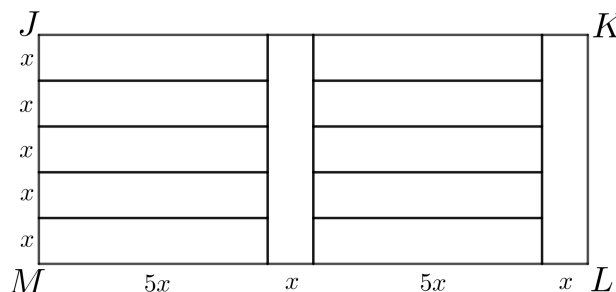## Problem C and Solution
A Rectangle of Rectangles

Problem

Large rectangle $JKLM$ is formed by twelve identical smaller rectangles, as shown.

If the area of $JKLM$ is 540 cm^2, then determine the dimensions of the smaller rectangles.

Solution

Let x be the width of one of the smaller identical rectangles, in cm. Five of the smaller rectangles are stacked on top of each other forming JM, so $JM = x + x + x + x + x = 5x$. Since $JKLM$ is a rectangle, $JM = KL = 5x$. Thus, $5x$ is also the length of a smaller rectangle. Therefore, a smaller rectangle is $5x$ cm by x cm.

From here, we proceed with two different solutions.

Solution 1

Since $JKLM$ is formed by twelve identical smaller rectangles, the area of rectangle $JKLM$ is equal to 12 times the area of one of the smaller rectangles.

$$\text{Area } JKLM = 12 \times \text{Area of one smaller rectangle}$$
$$540 = 12 \times 5x \times x$$
$$540 = 60 \times x^2$$

Dividing both sides by 60, we obtain $x^2 = 9$. Since x is the width of a smaller rectangle, $x > 0$, and so $x = 3$ follows.

Thus, the width of a smaller rectangle is $x = 3$ cm and the length of a smaller rectangle is $5x = 5(3) = 15$ cm.

Therefore, the smaller rectangles are each 15 cm by 3 cm.

Solution 2

Side length ML is made up of the lengths of two of the smaller rectangles plus the widths of two of the smaller rectangles. Therefore, $ML = 5x + 5x + x + x = 12x$ and rectangle $JKLM$ is $12x$ cm by $5x$ cm.

To find the area of $JKLM$ we multiply the length ML by the width JM.

$$\text{Area } JKLM = ML \times JM$$
$$540 = (12x) \times (5x)$$
$$540 = 12 \times 5 \times x \times x$$
$$540 = 60 \times x^2$$

Dividing both sides by 60, we obtain $x^2 = 9$. Since x is the width of a smaller rectangle, $x > 0$, and so $x = 3$ follows.

Thus, the width of a smaller rectangle is $x = 3$ cm and the length of a smaller rectangle is $5x = 5(3) = 15$ cm.

Therefore, the smaller rectangles are each 15 cm by 3 cm.
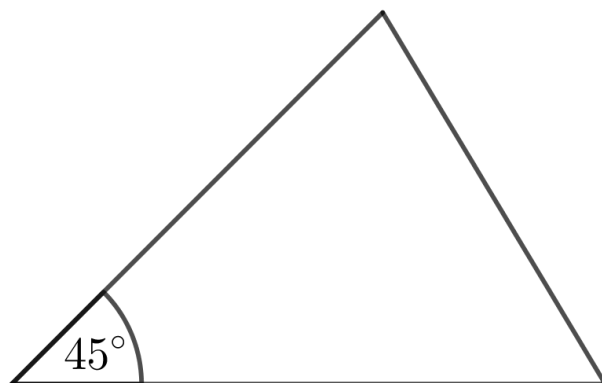
Problème de la semaine
Problème C
Deux angles sur trois

En mesurant les angles d'un triangle, Patricia a trouvé que l'un des angles mesure 45°. Elle a également remarqué que les mesures des deux autres angles du triangle sont dans un rapport de 4 : 5. Quelles sont les mesures de ces deux angles?
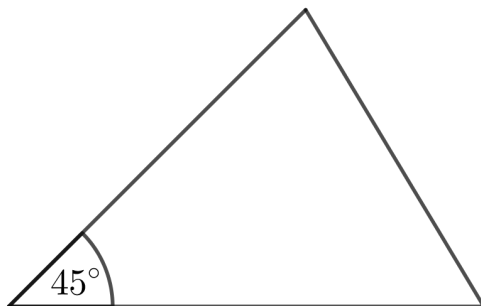

THÈMES : ALGÈBRE, GÉOMÉTRIE ET MESURE, SENS DU NOMBRE

Problem of the Week
Problem C and Solution
Two Out of Three Angles

Problem

While measuring the angles in a triangle, Patricia found the measure of one of the angles is $45°$. Once she had measured the other two angles, she noticed that the measures of these two angles are in the ratio $4 : 5$. What is the measure of each of the other two angles?

Solution

Solution 1

Since the measure of the unknown angles are in the ratio of $4 : 5$, then we can give the angles measures of $5n°$ and $4n°$.

Since the sum of the measures of the three angles in any triangle is $180°$, then

$$45 + 4n + 5n = 180$$
$$4n + 5n = 135$$
$$9n = 135$$
$$n = 15$$

Therefore, the other two angles are $5n = 5(15) = 75°$ and $4n = 4(15) = 60°$.

Solution 2

Since the sum of the measures of the three angles in any triangle is $180°$, then the sum of the measures of the two unknown angles in the triangle is
$180° - 45° = 135°$.

The measures of the two unknown angles are in the ratio $4 : 5$, and so one of the two angles measures $\frac{5}{4+5} = \frac{5}{9}$ of the sum of the two angles, while the other angle measures $\frac{4}{4+5} = \frac{4}{9}$ of the sum of the two angles.

That is, the larger of the two unknown angles measures $\frac{5}{9} \times 135° = 75°$, and the smaller of the unknown angles measures $\frac{4}{9} \times 135° = 60°$.

Therefore, the other two angles are $75°$ and $60°$.

Problème de la semaine
Problème C
Toujours plus haut

Priya a un contenant en forme de prisme rectangulaire, la base est de 40 cm sur 40 cm et la hauteur est 50 cm. Elle remplit le contenant avec de l'eau jusqu'à ce que l'eau atteigne la moitié de la hauteur du contenant. Un cube solide avec des côtés de 20 cm de longueur est déposé dans le contenant.

À quelle distance du dessus du contenant l'eau est-elle rendue maintenant ?

Problem of the Week
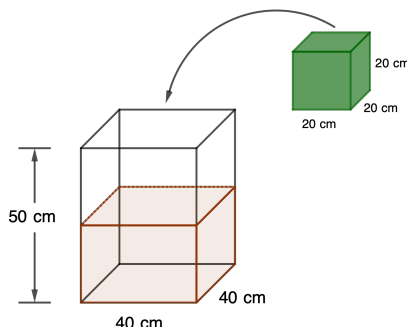## Problem C and Solution
Reach for the Top

Problem

Priya has a container in the shape of a rectangular prism with base 40 cm by 40 cm and height 50 cm. She fills the container with water so that the water reaches half of the height of the container. A solid cube with side length 20 cm is then placed in the container.

How far from the top of the container does the water now reach?

Solution

Solution 1

The height of the water is $\frac{50}{2} = 25$ cm. Since volume = length × width × height, the volume of water in the container is $40 \times 40 \times 25 = 40\,000$ cm^3.

The volume of the solid cube is $20 \times 20 \times 20 = 8000$ cm^3.

Thus, the total volume is $40\,000 + 8000 = 48\,000$ cm^3.

Let x represent the height of the water, in centimetres, after the cube is added. Then, using the formula for volume of a rectangular solid with 40 cm by 40 cm base, we have

$$48\,000 = 40 \times 40 \times x$$
$$48\,000 = 1600 \times x$$
$$x = 30$$

Therefore, the new water height is 30 cm and the water is $50 - 30 = 20$ cm from the top of the container.

Solution 2

Let h be the height of a rectangular prism with base 40 cm by 40 cm and with the same volume as the solid cube. Since volume = length × width × height, we have

$$40 \times 40 \times h = 20 \times 20 \times 20$$
$$1600 \times h = 8000$$
$$h = 5$$

Therefore, increasing the height of water in the container by 5 cm will increase the volume by 8000 cm^3, which is equal to the volume of the solid cube.

Therefore, the new water height is $25 + 5 = 30$ cm and the water is $50 - 30 = 20$ cm from the top of the container.

Problème de la semaine
Problème C
Tout autour

Deux segments de droite CE et CF sont perpendiculaires, chacun a une longueur de 10. Le rectangle $ABCD$ est tracé de manière à ce que D se situe sur CE, B se situe sur CF avec $BF = 4$ et la diagonale de $ABCD$ a une longueur de 10. Les segments de droite EA et AF sont ensuite tracés. Calcule le périmètre du quadrilatère $AFCE$, arrondis à une décimale près.
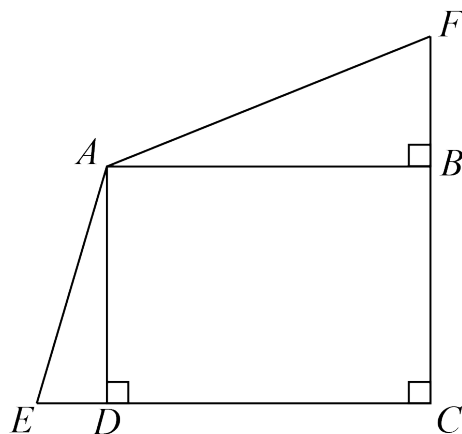
NOTE : Tu trouveras peut-être cette information utile :

La *théorème de Pythagore* stipule que "Dans un triangle rectangle, le carré de la longueur de l'hypothénuse (le côté opposé à l'angle droit) égale la somme des carrés des longueurs des deux côtés."

Par exemple, si c est la longueur de l'hypoténuse et a et b sont les longueurs des autres côtés, alors $c^2 = a^2 + b^2$.

THÈME : GÉOMÉTRIE ET MESURE
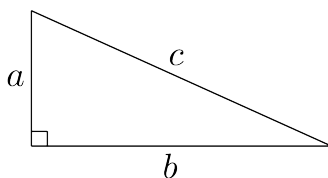
Problem of the Week
Problem C and Solution
Around the Outside

Problem

Two line segments, CE and CF, are perpendicular to each other, each with length 10. Rectangle $ABCD$ is drawn so that D is on CE, B is on $C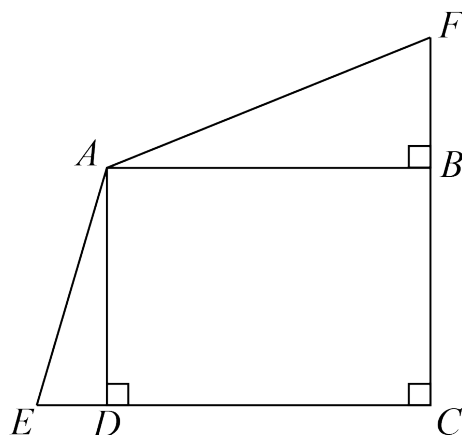F$ with $BF = 4$, and the diagonal of $ABCD$ has length 10. Line segments EA and AF are then drawn. Determine the perimeter of quadrilateral $AFCE$, rounded to one decimal place.

NOTE: You may find the following useful:

The *Pythagorean Theorem* states, "In a right-angled triangle, the square of the length of hypotenuse (the side opposite the right angle) equals the sum of the squares of the lengths of the other two sides."

For example, if c is the length of the hypotenuse, and a and b are the lengths of the other two sides, then $c^2 = a^2 + b^2$.

Solution

First, $BC = CF - BF = 10 - 4 = 6$. Since $ABCD$ is a rectangle, $AD = BC = 6$. We then use the Pythagorean Theorem in $\triangle ABC$.

$$
\begin{aligned}
AB^2 &= AC^2 - BC^2 \\
&= 10^2 - 6^2 \\
&= 100 - 36 \\
&= 64
\end{aligned}
$$

Therefore $AB = 8$, since $AB > 0$. Since $ABCD$ is a rectangle, $CD = AB = 8$. Then $DE = CE - CD = 10 - 8 = 2$. These lengths are shown on the diagram.
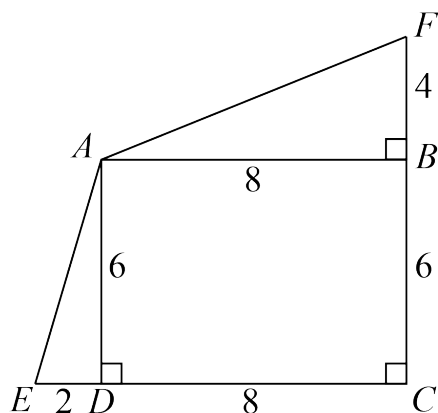
We then use the Pythagorean Theorem in $\triangle ADE$.

$$
\begin{aligned}
AE^2 &= AD^2 + DE^2 \\
&= 6^2 + 2^2 \\
&= 36 + 4 \\
&= 40
\end{aligned}
$$

Therefore $AE = \sqrt{40}$, since $AE > 0$.

We then use the Pythagorean Theorem in $\triangle ABF$.

$$
\begin{aligned}
AF^2 &= AB^2 + BF^2 \\
&= 8^2 + 4^2 \\
&= 64 + 16 \\
&= 80
\end{aligned}
$$

Therefore $AF = \sqrt{80}$, since $AF > 0$.

Thus, the perimeter of $AFCE$ is equal to
$AF + CF + CE + AE = \sqrt{80} + 10 + 10 + \sqrt{40} \approx 35.3$.

Problème de la semaine
Problème C
Un problème anguleux

Les segments de droite AB et CD sont parallèles, AB étant au-dessus de CD. Le point J est situé au-dessus de AB. Les points E et F sont situés sur CD, E étant à la gauche de F, de telle sorte que JE intersecte avec AB au point G et JF intersecte avec AB au point H.
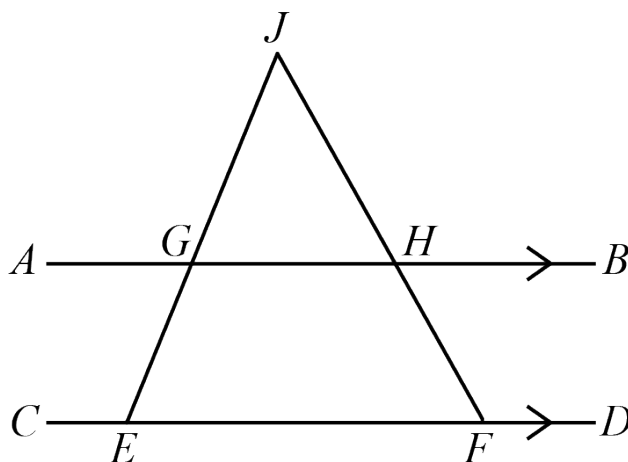
Si $\angle CEG = 110°$ et $\angle GHF = 122°$ trouve la mesure de $\angle GJH$.

Problem of the Week
Problem C and Solution
Angled

Problem

Line segments AB and CD are parallel, with AB above CD. Point J lies above AB, and points E and F lie on CD, with E to the left of F, so that JE intersects AB at G and JF intersects AB at H.
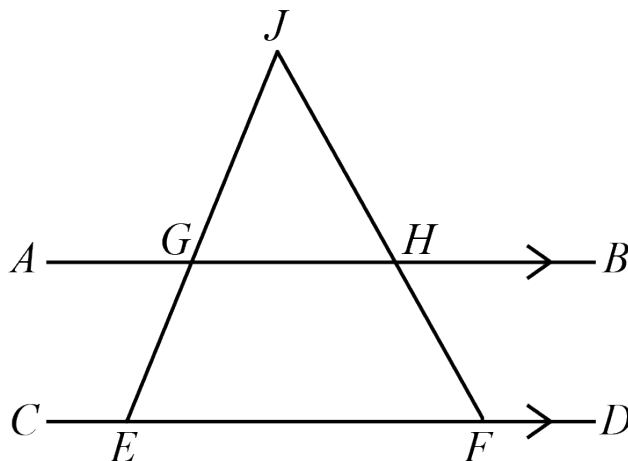
If $\angle CEG = 110°$ and $\angle GHF = 122°$, determine the measure of $\angle GJH$.

Solution

Solution 1

Since JHF is a straight line, then $\angle JHG = 180° - \angle GHF = 180° - 122° = 58°$. Since AB and CD are parallel, $\angle AGJ = \angle CEG = 110°$. Since AGH is a straight line, $\angle JGH = 180° - \angle AGJ = 180° - 110° = 70°$. Since the three angles in a triangle add to $180°$, then

$$\angle GJH = 180° - \angle JGH - \angle JHG = 180° - 70° - 58° = 52°.$$

Solution 2

Since JHF is a straight line, then $\angle JHG = 180° - \angle GHF = 180° - 122° = 58°$. Since AB and CD are parallel, $\angle JFE = \angle JHG = 58°$. Since CEF is a straight line, $\angle JEF = 180° - \angle CEG = 180° - 110° = 70°$. Since the three angles in a triangle add to $180°$, then

$$\angle GJH = \angle EJF = 180° - \angle JEF - \angle JFE = 180° - 70° - 58° = 52°.$$

NOTE: Since $\triangle GJH$ and $\triangle EJF$ have all three angles in common, we can say that they are *similar triangles*. Similar triangles have properties that make them very useful in geometry problems.

Problème de la semaine
Problème C
D'un bord à l'autre

Les segments de droite BP et AP se touchent au point P de sorte que $\angle BPA = 12°$. Les points Q, R, S, T, ... alternent d'un côté de l'angle à l'autre, avec Q sur le segment BP et R sur le segment AP, de telle sorte que chaque point se trouve à une plus grande distance du point P que le point précédent, et $PQ = QR = RS = ST = \dots$.
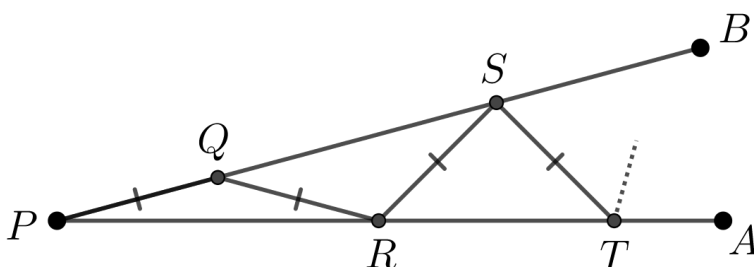
Cela crée les triangles isocèles $\triangle PQR$, $\triangle QRS$, $\triangle RST$ et ainsi de suite. Éventuellement, un des triangles isocèles sera également un triangle équilatéral.

Combien de triangles isocèles seront créés avant qu'un triangle équilatéral soit formé ?

THÈME : GÉOMÉTRIE ET MESURE
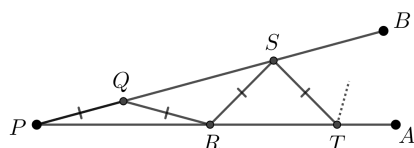
Problem of the Week
Problem C and Solution
Back and Forth

Problem

Line segments BP and AP are such that $\angle BPA = 12°$. Points Q, R, S, T, ... alternate from one arm of the angle to the other, with Q on BP and R on AP, such that each point is located farther away from P than the point before, and $PQ = QR = RS = ST = \ldots$. This creates isosceles triangles $\triangle PQR$, $\triangle QRS$, $\triangle RST$, and so on. Eventually, one of the isosceles triangles will also be an equilateral triangle.

How many isosceles triangles will be created before the equilateral triangle is created?

Solution

We will show that there will be 4 isosceles triangles created before the equilateral triangle is created. Throughout this solution there will be references to the Exterior Angle Theorem, which states: "The measure of an exterior angle of a triangle is equal to the sum of the measures of the opposite interior angles." For example, $\angle SQR$ is exterior to $\triangle QPR$ and $\angle SQR = \angle QPR + \angle QRP$.

We extend the diagram to include U on BP, V on AP, and W on BP such that $PQ = QR = RS = ST = TU = UV = VW$.
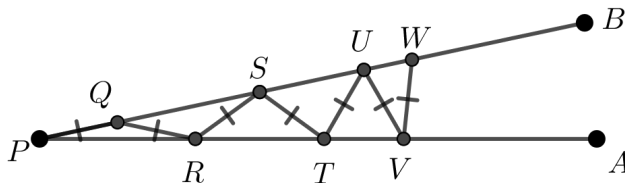
In $\triangle QPR$, $\angle QPR = 12°$ and $PQ = QR$. Therefore, $\triangle QPR$ is isosceles and $\angle QRP = \angle QPR = 12°$. Since $\angle SQR$ is exterior to $\triangle QPR$, by the Exterior Angle Theorem, $\angle SQR = \angle QPR + \angle QRP = 12° + 12° = 24°$.

In $\triangle RQS$, $\angle SQR = 24°$ and $QR = RS$. Therefore, $\triangle RQS$ is isosceles and $\angle RSQ = \angle SQR = 24°$. Since $\angle SRT$ is exterior to $\triangle PRS$, by the Exterior Angle Theorem, $\angle SRT = \angle SPR + \angle PSR = 12° + 24° = 36°$.

In $\triangle SRT$, $\angle SRT = 36°$ and $SR = ST$. Therefore, $\triangle SRT$ is isosceles and $\angle STR = \angle SRT = 36°$. Since $\angle UST$ is exterior to $\triangle PST$, by the Exterior Angle Theorem, $\angle UST = \angle SPT + \angle STP = 12° + 36° = 48°$.

In $\triangle TSU$, $\angle UST = 48°$ and $ST = TU$. Therefore, $\triangle TSU$ is isosceles and $\angle TUS = \angle UST = 48°$. Since $\angle UTV$ is exterior to $\triangle PUT$, by the Exterior Angle Theorem, $\angle UTV = \angle UPT + \angle PUT = 12° + 48° = 60°$.

In $\triangle UTV$, $\angle UTV = 60°$ and $TU = UV$. Therefore, $\triangle UTV$ is isosceles and $\angle UVT = \angle UTV = 60°$. Since $\angle UVT = \angle UTV = 60°$, the remaining angle in $\triangle UTV$ is $\angle TUV = 180° - 60° - 60° = 60°$. Since all three angles equal $60°$, $\triangle UTV$ is equilateral.

Since $\triangle UTV$ is the fifth isosceles triangle created, there are 4 isosceles triangles created before the equilateral triangle is created.

EXTENSION: In our problem, we started with $\angle APB = 12°$ and we eventually created an equilateral triangle. What other values for $\angle APB$ will eventually create an equilateral triangle?

Problème de la semaine
Problème C
Un trottoir mouillé

Un arroseur lance de l'eau sur une région circulaire dont le rayon est de 5 m. Il est positionné de façon à ce qu'une partie de l'eau atterrit sur le trottoir voisin.

Sur le diagramme, le cercle de centre O représente la région circulaire couverte par l'eau de l'arroseur. Les points A et B sont situés sur le cercle avec $\angle AOB = 90°$ et $OA = 5$ m. La région ombrée, qui est la partie située à l'intérieur du secteur AOB mais à l'extérieur du $\triangle AOB$, représente la zone du trottoir où l'eau atterrit.

Détermine l'aire de cette région du trottoir, arrondie à une décimale près.
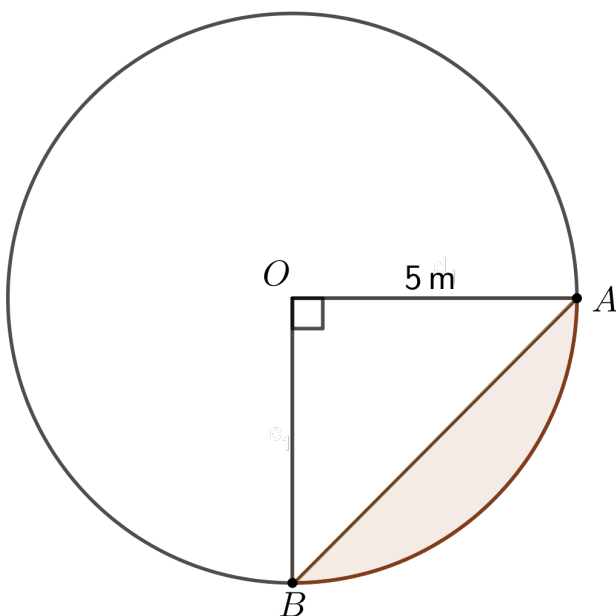
Problem of the Week
Problem C and Solution
A Wet Sidewalk

Problem

A sprinkler sprays water in a circular region with a radius of 5 m. It is positioned in such a way that some of the water is landing on a nearby sidewalk.

In the diagram, the circle with centre O represents the circular region covered by the water sprinkler. Points A and B lie on the circle with $\angle AOB = 90°$ and $OA = 5$ m. The shaded region, which is the region inside sector AOB but outside of $\triangle AOB$, represents the region of the sidewalk where water is landing.

Determine the area of the region of the sidewalk where the water is landing, correct to one decimal place.
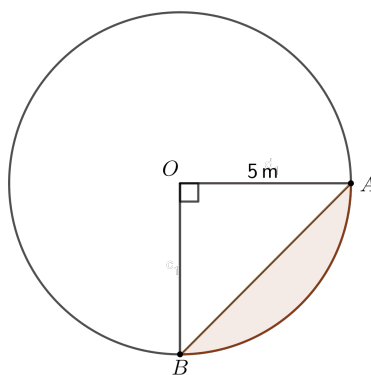
Solution

$\triangle AOB$ along with the shaded region cover $\frac{1}{4}$ of the area of the circle covered by the sprinkler. To determine the area of the shaded region, we need to find the area of the triangle and subtract it from one-quarter of the area of the circle.

Since $\angle AOB = 90°$, we can use OA as the base and OB as the height in the formula for the area of a triangle to find the area of $\triangle AOB$. Further, since OA and OB are radii of the circle, we know that $OA = OB = 5$ m. Therefore,

$$\text{Area } \triangle AOB = \frac{\text{base} \times \text{height}}{2} = \frac{OA \times OB}{2} = \frac{5 \times 5}{2} = \frac{25}{2} = 12.5 \text{ m}^2$$

To determine the area of the quarter circle, we will use the formula for the area of a circle, $A = \pi r^2$, and then divide the result by 4. That is,

$$\text{Area of the Quarter Circle} = \frac{\pi \times r^2}{4} = \frac{\pi \times 5 \times 5}{4} = \frac{\pi \times 25}{4} = (6.25 \times \pi) \text{ m}^2$$

We can now determine the area of the shaded region.

$$\begin{aligned}\text{Area of Shaded Region } &= \text{Area of the Quarter Circle} - \text{Area } \triangle AOB \\ &= 6.25 \times \pi - 12.5 \\ &\approx 7.1 \text{ m}^2\end{aligned}$$

Therefore, the area of the shaded region, correct to one decimal place, is 7.1 m^2.

Problème de la semaine
Problème C
Trois triangles

Le triangle rectangle $\triangle ABC$ comprend $\angle ABC = 90°$ et $AB = 12$. Le point D est situé sur le côté BC tel que $BD = 5$. L'aire du $\triangle ADC$ représente 80% de l'aire du $\triangle ABD$.
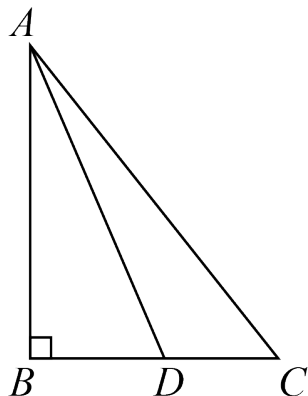
Détermine le périmètre du $\triangle ADC$.

REMARQUE: Tu trouveras peut-être ce qui suit utile :

Le *Théorème de Pythagore* stipule que: « Dans un triangle rectangle, le carré de la longueur de l'hypoténuse (le côté opposé à l'angle droit) est égal à la somme des carrés des longueurs des deux autres côtés du triangle. »

Dans le triangle rectangle représenté, c est l'hypoténuse, a et b sont les longueurs des deux autres côtés et $c^2 = a^2 + b^2$.

THÈME : GÉOMÉTRIE ET MESURE
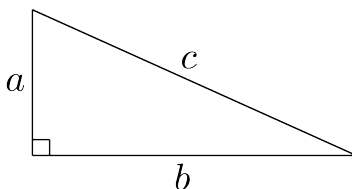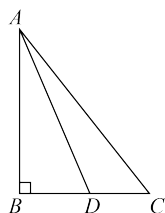
Problem of the Week
Problem C and Solution
Three Triangles

Problem

The right-angled triangle $\triangle ABC$ has $\angle ABC = 90°$ and $AB = 12$. Point D is on side BC such that $BD = 5$ and the area of $\triangle ADC$ is 80% of the area of $\triangle ABD$. Determine the perimeter of $\triangle ADC$.

Solution

We start by labeling our diagram with the given information. We then notice that since AB is perpendicular to BC, it follows that AB is perpendicular to DC. Thus, if the base of $\triangle ADC$ is DC, then its height is AB.

To find the area of a triangle, multiply the length of the base by the height and divide by 2. Therefore,

$$\text{area of } \triangle ABD = BD \times AB \div 2 = 5 \times 12 \div 2 = 30.$$

Since the area of $\triangle ADC$ is 80% of the area of $\triangle ABD$, the area of $\triangle ADC$ is equal to $0.8 \times 30 = 24$.

We also know the area of $\triangle ADC$ is equal to $DC \times AB \div 2$. Thus,

$$DC \times AB \div 2 = 24$$
$$DC \times 12 \div 2 = 24$$
$$DC \times 6 = 24$$
$$DC = 24 \div 6 = 4$$

Thus, $BC = BD + DC = 5 + 4 = 9$. Using the Pythagorean Theorem in $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$
$$= 12^2 + 9^2$$
$$= 225$$

Thus, $AC = \sqrt{225} = 15$, since $AC > 0$.

Using the Pythagorean Theorem in $\triangle ABD$,

$$AD^2 = AB^2 + BD^2$$
$$= 12^2 + 5^2$$
$$= 169$$

Thus, $AD = \sqrt{169} = 13$, since $AD > 0$.

Therefore, the perimeter of $\triangle ADC$ is $AD + DC + AC = 13 + 4 + 15 = 32$.

Problème de la semaine
Problème C
Carrelage efficace

Sophia a une réserve illimitée de carreaux carrés. Elle possède des carreaux de 1 cm sur 1 cm, 2 cm sur 2 cm, 3 cm sur 3 cm, et ainsi de suite. Chaque carreau a une longueur de côté exprimée en un nombre entier de centimètres.

Un plancher de salle de bain rectangulaire, d'une aire de 84 cm sur 140 cm, doit être recouvert de carreaux carrés de dimension identique, sans qu'on doive les couper. Sophia sait que le plancher peut être recouvert par 11 760 carreaux de 1 cm sur 1 cm, puisque $84 \times 140 = 11\,760$ cm^2. Cependant, elle souhaite utiliser le plus petit nombre possible de carreaux de même taille pour accomplir cette tâche.

Détermine le nombre minimal de carreaux de taille identique nécessaires pour recouvrir complètement le plancher de la salle de bain.

THÈMES : GÉOMÉTRIE ET MESURE, SENS DU NOMBRE

Problem of the Week
Problem C and Solution
Efficient Tiling

Problem

Sophia has an unlimited supply of square tiles. Sophia has 1 cm by 1 cm tiles, 2 cm by 2 cm tiles, 3 cm by 3 cm tiles, and so on. Every tile has integer side lengths, in centimetres.

A rectangular bathroom floor with an 84 cm by 140 cm surface is to be completely covered by identical square tiles, none of which can be cut. Sophia knows that the floor can be completely covered with 11 760 tiles of size 1 cm by 1 cm, since $84 \times 140 = 11\,760$ cm^2. However, Sophia wants to use the minimum number of identical sized tiles to complete the job.

Determine the minimum number of identical sized tiles required to completely cover the bathroom floor.

Solution

To use the least number of tiles, Sophia must use the largest tile possible. The square tile must have sides less than or equal to 84 cm. If the tile has side length greater than 84 cm, it would have to be cut to fit the width of the floor.

Since the tiles are square and must completely cover the floor, the side length of the tile must be a number that is a factor of both 84 and 140. In fact, since Sophia wants the largest side length, she is looking for the greatest common factor of 84 and 140.

The positive factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, and 84.

The positive factors of 140 are 1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70, and 140.

The largest number common to both lists is 28. Therefore, the greatest common factor of 84 and 140 is 28. The required tiles are 28 cm by 28 cm. Since $84 \div 28 = 3$, the surface is 3 tiles wide. Since $140 \div 28 = 5$, the surface is 5 tiles long. The minimum number of tiles required is $3 \times 5 = 15$ tiles.

The number of 28 cm by 28 cm tiles required to cover the floor area is 15. This is the minimum number of tiles required.

Problème de la semaine
Problème C
Une aire irrégulière

Les points $A(7, 11)$, $B(9, 9)$, $C(6, 2)$ et $D(2, 6)$ forment le quadrilatère $ABCD$.
Détermine l'aire du quadrilatère $ABCD$.
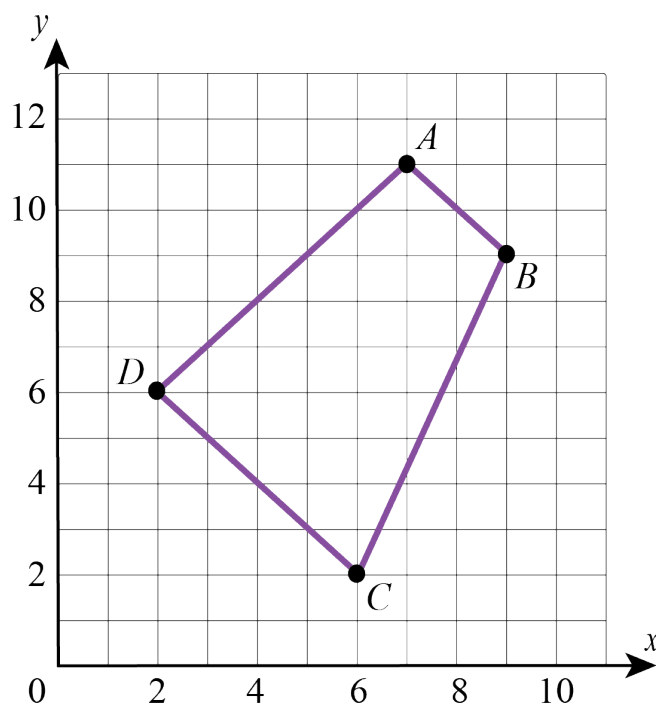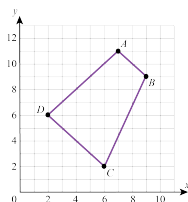
Problem of the Week
Problem C and Solution
Irregular Area

Problem

Points $A(7, 11)$, $B(9, 9)$, $C(6, 2)$, and $D(2, 6)$ form quadrilateral $ABCD$. Determine the area of $ABCD$.

Solution
Solution 1

Consider points $E(2, 11)$, $F(9, 11)$, $G(9, 2)$, and $H(2, 2)$. Draw in $EFGH$.
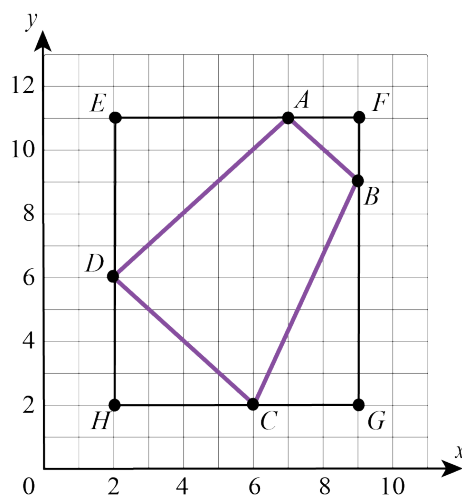
Since E and F both have y-coordinate 11, EF is a horizontal line which passes through A.

Since F and G both have x-coordinate 9, FG is a vertical line which passes through B.

Since G and H both have y-coordinate 2, GH is a horizontal line which passes through C.

Since E and H both have x-coordinate 2, EH is a vertical line which passes through D.

Thus, $EFGH$ is a rectangle that encloses $ABCD$. Also,

$$\text{area } ABCD = \text{area } EFGH - \text{area } \triangle AED - \text{area } \triangle AFB - \text{area } \triangle BGC - \text{area } \triangle CHD$$

In rectangle $EFGH$, $EF = 9 - 2 = 7$ and $EH = 11 - 2 = 9$. The area of rectangle $EFGH = EF \times EH = 7 \times 9 = 63$ units^2.

Since $EFGH$ is a rectangle, $\triangle AED$ is right-angled at E. Since $AE = 7 - 2 = 5$ and $ED = 11 - 6 = 5$, the area of $\triangle AED = \frac{AE \times ED}{2} = \frac{5 \times 5}{2} = \frac{25}{2} = 12.5$ units^2.

Since $EFGH$ is a rectangle, $\triangle AFB$ is right-angled at F. Since $FA = 9 - 7 = 2$ and $FB = 11 - 9 = 2$, the area of $\triangle AFB = \frac{FA \times FB}{2} = \frac{2 \times 2}{2} = \frac{4}{2} = 2$ units^2.

Since $EFGH$ is a rectangle, $\triangle BGC$ is right-angled at G. Since $GC = 9 - 6 = 3$ and $BG = 9 - 2 = 7$, the area of $\triangle BGC = \frac{GC \times BG}{2} = \frac{3 \times 7}{2} = \frac{21}{2} = 10.5$ units^2.

Since $EFGH$ is a rectangle, $\triangle CHD$ is right-angled at H. Since $CH = 6 - 2 = 4$ and $DH = 6 - 2 = 4$, the area of $\triangle CHD = \frac{CH \times DH}{2} = \frac{4 \times 4}{2} = \frac{16}{2} = 8$ units^2.

Therefore,

$$\begin{aligned}
\text{area } ABCD &= \text{area } EFGH - \text{area } \triangle AED - \text{area } \triangle AFB - \text{area } \triangle BGC - \text{area } \triangle CHD \\
&= 63 - 12.5 - 2 - 10.5 - 8 \\
&= 30 \text{ units}^2
\end{aligned}$$

Solution 2

Consider points $E(6, 10)$, $F(7, 9)$, $G(6, 6)$, and $H(6, 9)$. Draw in AF, EC, HB, and DG.
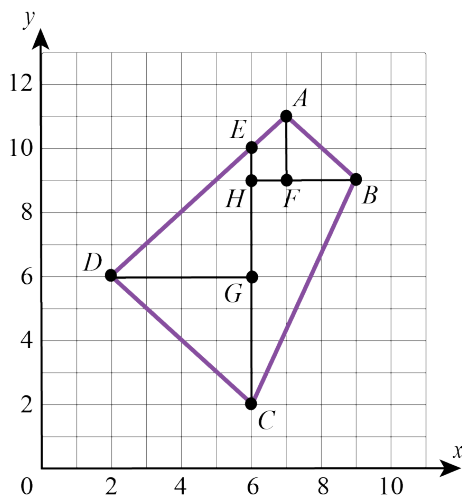
Since A and F both have x-coordinate 7, AF is a vertical line.

Since E, H, G, and C have x-coordinate 6, EC is a vertical line which passes through H and G.

Since H, F, and B have y-coordinate 9, HB is a horizontal line which passes through F.

Since D and G both have y-coordinate 6, DG is a horizontal line.

To determine the area of $ABCD$, we find the areas in the interior shapes, $\triangle AFB$, $\triangle BHC$, $\triangle DGC$, $\triangle DGE$, and trapezoid $EAFH$, and calculate their sum.

Since AF is vertical and HB is horizontal and passes through F, $\triangle AFB$ is right-angled at F. Since $AF = 11 - 9 = 2$ and $FB = 9 - 7 = 2$, area $\triangle AFB = \frac{AF \times FB}{2} = \frac{2 \times 2}{2} = \frac{4}{2} = 2$ units^2.

Since HB is horizontal and EC is vertical and passes through H, $\triangle BHC$ is right-angled at H. Since $HB = 9 - 6 = 3$ and $HC = 9 - 2 = 7$, area $\triangle BHC = \frac{HB \times HC}{2} = \frac{3 \times 7}{2} = \frac{21}{2} = 10.5$ units^2.

Since DG is horizontal and EC is vertical and passes through G, $\triangle DGC$ is right-angled at G. Since $DG = 6 - 2 = 4$ and $GC = 6 - 2 = 4$, area $\triangle DGC = \frac{DG \times GC}{2} = \frac{4 \times 4}{2} = \frac{16}{2} = 8$ units^2.

Since DG is horizontal and EC is vertical and passes through G, $\triangle DGE$ is right-angled at G. Since $DG = 6 - 2 = 4$ and $EG = 10 - 6 = 4$, area $\triangle DGE = \frac{DG \times EG}{2} = \frac{4 \times 4}{2} = \frac{16}{2} = 8$ units^2.

Since trapezoid $EAFH$ has side lengths $EH = 10 - 9 = 1$ and $AF = 11 - 9 = 2$, and height $HF = 7 - 6 = 1$, area trapezoid $EAFH = \frac{EH + AF}{2} \times HF = \frac{1+2}{2} \times 1 = \frac{3}{2} \times 1 = 1.5$ units^2.

Therefore,

$$\begin{aligned}
\text{area } ABCD &= \text{area } \triangle AFB + \text{area } \triangle BHC + \text{area } \triangle DGC + \text{area } \triangle DGE + \text{area trapezoid } \triangle EAFH \\
&= 2 + 10.5 + 8 + 8 + 1.5 \\
&= 30 \text{ units}^2
\end{aligned}$$

Sens du nombre (N)

Amène-moi à la couverture

Problème de la semaine
Problème C
Colocataires aquatiques

Niall a trois grands aquariums et il aimerait mettre des tétras et des guppys dans chaque aquarium. Il a 19 tétras et 18 guppys et il ne veut pas qu'il y ait plus de guppys que de tétras dans chaque aquarium. Niall aimerait également que chaque aquarium contienne au moins 5 tétras et 3 guppys.

Détermine le nombre maximum de poissons que peut contenir l'un de ses aquariums.

Problem of the Week
Problem C and Solution
Tankmates

Problem

Niall has three large fish tanks and would like to put some tetras and guppies in each tank. He currently has 19 tetras and 18 guppies and doesn't want any of his tanks to contain more guppies than tetras. Also, he would like each tank to have at least 5 tetras and 3 guppies.

Determine the largest number of fish that can be in one of his fish tanks.

Solution

Since each fish tank must have at least 5 tetras and 3 guppies, we will start by placing 5 tetras and 3 guppies into each fish tank. The number of fish now in each tank is summarized in the table below.

	Tank 1	Tank 2	Tank 3
Number of tetras	5	5	5
Number of guppies	3	3	3

We have placed $5 \times 3 = 15$ tetras and $3 \times 3 = 9$ guppies, so we have $19 - 15 = 4$ tetras and $18 - 9 = 9$ guppies left to place.

In order to place the largest number of fish in a tank, as many as possible of the remaining fish should be placed in one particular tank. We will start by adding all of the remaining tetras to Tank 1. The number of fish now in each tank is summarized in the table below.

	Tank 1	Tank 2	Tank 3
Number of tetras	9	5	5
Number of guppies	3	3	3

Since Niall doesn't want any of his tanks to contain more guppies than tetras, Tank 1 can contain at most 9 guppies. Since we want Tank 1 to contain the largest number of fish, we will add 6 guppies to Tank 1.

Then we have $9 - 6 = 3$ guppies left to place. We cannot add them to Tank 1. However, we can add 2 guppies to Tank 2, and 1 guppy to Tank 3. The number of fish now in each tank is summarized in the table below.

	Tank 1	Tank 2	Tank 3
Number of tetras	9	5	5
Number of guppies	9	5	4

Therefore, the largest number of fish that can be in one of Niall's fish tanks is 18.

Problème de la semaine
Problème C
À chacun son gâteau

Jessica, Callista, Peter et Monica sont allés à la boulangerie pour acheter sept gâteaux. Chaque gâteau coûte 9,00 $. Jessica a payé 27,00 $, Callista a payé 9,00 $ et Peter a payé 22,50 $. Monica a payé le montant restant. Ils partagent les gâteaux de manière que la fraction du montant total que chaque personne a payée corresponde à la fraction de la quantité totale de gâteau que chaque personne reçoit.

Quelle quantité de gâteau chaque personne devrait-elle recevoir?

Problem of the Week
Problem C and Solution
They Take the Cake

Problem

Jessica, Callista, Peter, and Monica went to the bakery to buy seven cakes. Each cake costs $9.00. Jessica paid $27.00, Callista paid $9.00, and Peter paid $22.50. Monica paid the remaining amount. They divide the cakes so that the fraction of the total that each person paid is equal to the fraction of the total amount of cake that each person receives.

What amount of cake should each person receive?

Solution
Solution 1

If Jessica, Callista, Peter and Monica buy a total of seven cakes at $9.00 per cake, then they paid a total of $7 \times \$9.00 = \$63.00$.

Jessica paid $27.00, Callista paid $9.00, Peter paid $22.50 and Monica paid the remainder. Therefore, Monica paid $\$63.00 - \$27.00 - \$9.00 - \$22.50 = \$4.50$.

Since $3 \times \$9 = \$27.00$ then Jessica should receive 3 cakes.

Since $1 \times \$9 = \$9.00$ then Callista should receive 1 cake.

Since $0.5 \times \$9 = \$4.50$ then Monica should receive 0.5 cakes.

Thus, Peter should get $7 - 3 - 1 - 0.5 = 2.5$ cakes.

Solution 2

If Jessica, Callista, Peter and Monica buy a total of seven cakes at $9.00 per cake, then they paid a total of $7 \times \$9.00 = \$63.00$.

Jessica paid $27.00, Callista paid $9.00, Peter paid $22.50 and Monica paid the remainder. Therefore, Monica paid $\$63.00 - \$27.00 - \$9.00 - \$22.50 = \$4.50$.

Jessica should receive $\frac{27}{63} = \frac{3}{7}$ of the total amount of cake. So Jessica should receive $\frac{3}{7}$ of 7, or $\frac{3}{7} \times 7 = 3$ cakes.

Callista should receive $\frac{9}{63} = \frac{1}{7}$ of the total amount of cake. So Callista should receive $\frac{1}{7}$ of 7, or $\frac{1}{7} \times 7 = 1$ cake.

Peter should receive $\frac{22.5}{63} = \frac{225}{630} = \frac{25}{70} = \frac{5}{14}$ of the total amount of cake. So Peter should receive $\frac{5}{14}$ of 7, or $\frac{5}{14} \times 7 = \frac{5}{2} = 2\frac{1}{2}$ cakes.

Monica should receive $\frac{4.50}{63} = \frac{45}{630} = \frac{5}{70} = \frac{1}{14}$ of the total amount of cake. So Monica should receive $\frac{1}{14}$ of 7, or $\frac{1}{14} \times 7 = \frac{1}{2}$ of a cake.

We can check that all of the cake has been distributed. If there were any errors in our solution, this may help us catch them. Jessica receives 3 cakes, Callista receives 1 cake, Peter receives $2\frac{1}{2}$ cakes and Monica receives $\frac{1}{2}$ of a cake. The total number of cakes distributed is $3 + 1 + 2\frac{1}{2} + \frac{1}{2} = 7$, as required.

Problème de la semaine
Problème C
La suite de l'année

Dans une suite de nombres, chaque nombre est appelé un *terme*. Dans la suite 2, 4, 6, 8, le premier terme est 2, le deuxième terme est 4, le troisième terme est 6 et le quatrième terme est 8.

Le premier terme d'une autre suite est 24. On peut déterminer les termes suivants de cette suite de la manière suivante:

— Si un terme est pair, on le divise par 2 pour obtenir le terme suivant.
— Si un terme est impair, on le multiplie par 3, puis on ajoute 1 au produit pour obtenir le terme suivant.

En procédant ainsi, on peut déterminer que les trois premiers termes de la suite sont 24, 12 et 6.

Quel est le 2024ᵉ terme de la suite?

Problem of the Week
Problem C and Solution
The Sequence of the Year

Problem

In a sequence of numbers, each number in the sequence is called a *term*. In the sequence 2, 4, 6, 8, the first term is 2, the second term is 4, the third term is 6, and the fourth term is 8.

In another sequence, the first term is 24. We can determine the next terms in the sequence as follows:

- If a term is even, then divide it by 2 to get the next term.

- If a term is odd, then multiply it by 3 and add 1 to get the next term.

By doing this, we can determine that the first three terms in the sequence are 24, 12, and 6. What is the 2024th term in the sequence?

Solution

We will begin by finding more terms in the sequence. The first 14 terms of the sequence are 24, 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1.

If we continue the sequence, we will see that the terms 4, 2, and 1 will continue to repeat. Thus, the 9th term, 12th term, 15th term, and so on, will each have a value of 4. Notice that these term numbers are all multiples of 3. It follows that every term number after 9 that is a multiple of 3 will have a value of 4.

Thus, since 2022 is a multiple of 3, the 2022nd term will have a value of 4. Then, the 2023rd term will have a value of 2, and the 2024th term will have a value of 1.

EXTENSION:

In 1937, the mathematician Lothat Collatz wondered if any sequence whose terms after the first are determined in this way would always eventually reach the number 1, regardless of which number you started with. This problem is actually still unsolved today and is called the Collatz Conjecture.

Problème de la semaine
Problème C
Deux angles sur trois

En mesurant les angles d'un triangle, Patricia a trouvé que l'un des angles mesure 45°. Elle a également remarqué que les mesures des deux autres angles du triangle sont dans un rapport de 4 : 5. Quelles sont les mesures de ces deux angles?

Problem of the Week
Problem C and Solution
Two Out of Three Angles

Problem

While measuring the angles in a triangle, Patricia found the measure of one of the angles is $45°$. Once she had measured the other two angles, she noticed that the measures of these two angles are in the ratio $4 : 5$. What is the measure of each of the other two angles?

Solution

Solution 1

Since the measure of the unknown angles are in the ratio of $4 : 5$, then we can give the angles measures of $5n°$ and $4n°$.

Since the sum of the measures of the three angles in any triangle is $180°$, then

$$45 + 4n + 5n = 180$$
$$4n + 5n = 135$$
$$9n = 135$$
$$n = 15$$

Therefore, the other two angles are $5n = 5(15) = 75°$ and $4n = 4(15) = 60°$.

Solution 2

Since the sum of the measures of the three angles in any triangle is $180°$, then the sum of the measures of the two unknown angles in the triangle is
$180° - 45° = 135°$.

The measures of the two unknown angles are in the ratio $4 : 5$, and so one of the two angles measures $\frac{5}{4+5} = \frac{5}{9}$ of the sum of the two angles, while the other angle measures $\frac{4}{4+5} = \frac{4}{9}$ of the sum of the two angles.

That is, the larger of the two unknown angles measures $\frac{5}{9} \times 135° = 75°$, and the smaller of the unknown angles measures $\frac{4}{9} \times 135° = 60°$.

Therefore, the other two angles are $75°$ and $60°$.

Problème de la semaine
Problème C
Cinq aimants

Harlow a cinq aimants, chacun portant un chiffre différent de 1 à 5. Elle a disposé ces aimants de manière à former un nombre de cinq chiffres, $ABCDE$, tel que:

— le nombre de trois chiffres ABC soit divisible par 4,

— le nombre de trois chiffres BCD soit divisible par 5 et

— le nombre de trois chiffres CDE soit divisible par 3.

Détermine le nombre de cinq chiffres que Harlow a formé.

12345

Problem of the Week
Problem C and Solution
Five Magnets

Problem

Harlow has five magnets, each with a different number from 1 to 5. They arranged these magnets to create a five digit number $ABCDE$ such that:

- the three-digit number ABC is divisible by 4,

- the three-digit number BCD is divisible by 5, and

- the three-digit number CDE is divisible by 3.

Determine the five-digit number that Harlow created.

Solution

Since ABC is divisible by 4, it follows that C must be even, so $C = 2$ or $C = 4$.

Since BCD is divisible by 5, it follows that $D = 0$ or $D = 5$. However, there is no magnet with a 0, so it follows that $D = 5$.

We also know that CDE is divisible by 3. We can consider the following two cases.

- **Case 1:** $C = 2$
  If $C = 2$, then the three-digit number CDE is $25E$. The only possibilities for E are 1, 3, or 4. However, none of 251, 253 and 254 are divisible by 3. It follows that C cannot equal 2.

- **Case 2:** $C = 4$
  If $C = 4$ then the three-digit number CDE is $45E$. The only possibilities for E are 1, 2, or 3. Since 451 and 452 are not divisible by 3, but 453 is divisible by 3, it follows that $C = 4$ and $E = 3$.

Thus, the three-digit number ABC is $AB4$. The only magnets not used yet are numbered 1 and 2, so this number is 124 or 214. Since 214 is not divisible by 4, but 124 is divisible by 4, it follows that $A = 1$ and $B = 2$.

Therefore, the five-digit number must be 12453.

Problème de la semaine
Problème C
Toujours plus haut

Priya a un contenant en forme de prisme rectangulaire, la base est de 40 cm sur 40 cm et la hauteur est 50 cm. Elle remplit le contenant avec de l'eau jusqu'à ce que l'eau atteigne la moitié de la hauteur du contenant. Un cube solide avec des côtés de 20 cm de longueur est déposé dans le contenant.

À quelle distance du dessus du contenant l'eau est-elle rendue maintenant ?

Problem of the Week
Problem C and Solution
Reach for the Top

Problem

Priya has a container in the shape of a rectangular prism with base 40 cm by 40 cm and height 50 cm. She fills the container with water so that the water reaches half of the height of the container. A solid cube with side length 20 cm is then placed in the container.

How far from the top of the container does the water now reach?

Solution

Solution 1

The height of the water is $\frac{50}{2} = 25$ cm. Since volume = length × width × height, the volume of water in the container is $40 \times 40 \times 25 = 40\,000$ cm^3.

The volume of the solid cube is $20 \times 20 \times 20 = 8000$ cm^3.

Thus, the total volume is $40\,000 + 8000 = 48\,000$ cm^3.

Let x represent the height of the water, in centimetres, after the cube is added. Then, using the formula for volume of a rectangular solid with 40 cm by 40 cm base, we have

$$48\,000 = 40 \times 40 \times x$$
$$48\,000 = 1600 \times x$$
$$x = 30$$

Therefore, the new water height is 30 cm and the water is $50 - 30 = 20$ cm from the top of the container.

Solution 2

Let h be the height of a rectangular prism with base 40 cm by 40 cm and with the same volume as the solid cube. Since volume = length × width × height, we have

$$40 \times 40 \times h = 20 \times 20 \times 20$$
$$1600 \times h = 8000$$
$$h = 5$$

Therefore, increasing the height of water in the container by 5 cm will increase the volume by 8000 cm^3, which is equal to the volume of the solid cube.

Therefore, the new water height is $25 + 5 = 30$ cm and the water is $50 - 30 = 20$ cm from the top of the container.

Problème de la semaine
Problème C
Tous les chiffres

Georgina a répertorié tous les nombres entiers de 1 à 13 :

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13$$

Elle a réalisé que la somme de tous les chiffres des nombres entiers dans cette liste est

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + (1 + 0) + (1 + 1) + (1 + 2) + (1 + 3) = 55$$

Elle te met au défi de trouver la somme de tous les chiffres des nombres entiers de 1 à 100. Quelle somme obtiendras-tu ?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

THÈME: SENS DU NOMBRE

Problem of the Week
Problem C and Solution
All the Digits

Problem

Georgina listed the integers from 1 to 13:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13$$

She determined that the sum of all of the digits of the integers in this list is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + (1 + 0) + (1 + 1) + (1 + 2) + (1 + 3) = 55$$

She then challenged you to determine the sum of all of the digits of the integers from 1 to 100. What sum would you get?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solution

In the integers from 1 to 100, each digit from 0 to 9 appears as the units digit of a number exactly ten times. For example, the digit 1 appears as the units digit in the numbers 1, 11, 21, 31, 41, 51, 61, 71, 81, 91, a total of 10 numbers. Therefore, the sum of all of the units digits is

$$10(1) + 10(2) + 10(3) + 10(4) + 10(5) + 10(6) + 10(7) + 10(8) + 10(9) + 10(0)$$
$$= 10(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 0)$$
$$= 10(45)$$
$$= 450$$

Similarly, each digit from 1 to 9 appears as the tens digit of a number exactly ten times. For example, the digit 1 appears as the tens digit in the numbers 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, a total of 10 numbers. The digit 0 appears as the tens digit of a number once. Therefore, the sum of all of the tens digits is

$$10(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) + 0 = 10(45)$$
$$= 450$$

The number 100 is the only number with a hundreds digit. We need to add 1 to our final sum.

Therefore, the sum of all of the digits of the integers from 1 to 100 is $450 + 450 + 1 = 901$.

Problème de la semaine
Problème C
Une erreur moyenne

Maggie est responsable de ramasser les œufs chaque jour à la ferme PDLS. Elle enregistre le nombre d'œufs ramassés chaque jour pour la durée d'une semaine. Ensuite, elle a calculé le nombre moyen d'œufs ramassés par jour, celui-ci était de 77. Lorsqu'elle a revérifié le nombre d'œufs qu'elle a enregistré pour chaque jour, elle a remarqué que le mercredi, elle avait enregistré 18 œufs. Toutefois, elle a réellement ramassé 81 œufs.

Quel est le nombre moyen d'œufs ramassés par jour pour cette semaine, une fois l'erreur corrigée ?

THÈME : Sens du nombre
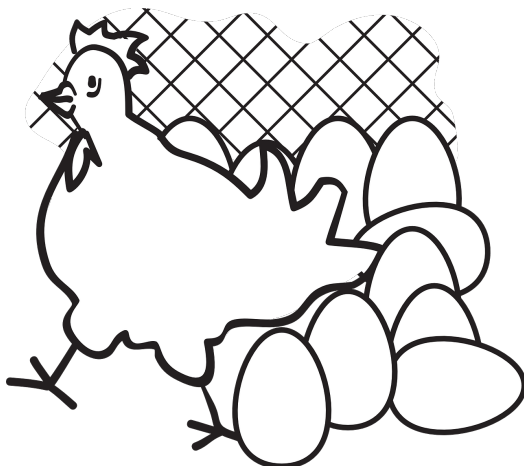
Problem of the Week
Problem C and Solution
An Average Error

Problem

At POTW Farms, Maggie is responsible for collecting eggs each day. She recorded the number of eggs collected each day for one week, and then calculated that the average number of eggs collected in one day that week was 77. When she double-checked the number of eggs she recorded for each day, she noticed that she had recorded 18 eggs on Wednesday. However, she had actually collected 81 eggs.

What is the corrected average number of eggs collected in one day for that week?

Solution
Solution 1

This first solution works primarily with the definition of an average.

To calculate the average, we add the recorded number of eggs for each of the seven days and divide by 7.

$$\frac{\text{sum of recorded number eggs for the seven days}}{7} = 77$$

To then obtain the sum of the recorded number of eggs collected for the seven days, we would multiply the average by 7.

$$\text{sum of recorded number of eggs for the seven days} = 7 \times 77 = 539$$

Now this sum includes the wrong number of eggs recorded on Wednesday of 18. So, we need to adjust the sum by subtracting the wrong number of eggs and adding the corrected number of eggs.

$$\text{correct sum of number of eggs for the seven days} = 539 - 18 + 81 = 602$$

We can now obtain Maggie's corrected average by dividing the corrected total by 7.

$$\text{correct average} = \frac{\text{correct sum of number of eggs for the seven days}}{7} = \frac{602}{7} = 86$$

Therefore, the corrected average is 86 eggs per day.

Solution 2

The second solution looks at how an increase in the number of eggs on one day will affect an overall average.

For an average based on seven days, an increase of 1 egg will cause the overall average to increase by $\frac{1}{7}$ of 1 egg. So, for each increase of 7 eggs, the overall average will increase by 1 egg. That is, the average number eggs will increase by 1 egg for every 7 eggs increased on Wednesday.

The number of eggs on Wednesday increases by $81 - 18 = 63$ eggs. Since $63 \div 7 = 9$, her average will increase by 9 eggs from 77 to 86 eggs.

Therefore, the corrected average is 86 eggs per day.

Problème de la semaine
Problème C
Quel est le reste ?

3^6 signifie $3 \times 3 \times 3 \times 3 \times 3 \times 3$ et est égal à 729 lorsqu'exprimé comme un nombre entier. Lorsque 3^6 est divisé par 5, le reste est 4.

Lorsque le nombre entier 3^{2025} est divisé par 5, quel est le reste ?

$$5 \overline{)\, 3^{2025}} \quad \text{Quotient}$$

Reste ?

$$5 \overline{) \, 3^{2025}}$$ Quotient

Remainder?

Problem of the Week
Problem C and Solution
What is the Remainder?

Problem

3^6 means $3 \times 3 \times 3 \times 3 \times 3 \times 3$ and is equal to 729 when expressed as an integer. When 3^6 is divided by 5, the remainder is 4.

When the integer 3^{2025} is divided by 5, what is the remainder?

Solution

When the units digit of an integer is 0 or 5, the remainder is 0 when divided by 5. When the units digit of an integer is 1 or 6, the remainder is 1 when divided by 5. For example, $56 \div 5 = 11$ R1. When the units digit of an integer is 2 or 7, the remainder is 2 when divided by 5. When the units digit of an integer is 3 or 8, the remainder is 3 when divided by 5. When the units digit of an integer is 4 or 9, the remainder is 4 when divided by 5.

Let's examine the pattern of the units digits of the first eight powers of 3.

Power	Value	Units Digit
3^1	3	3
3^2	9	9
3^3	27	7
3^4	81	1
3^5	243	3
3^6	729	9
3^7	2187	7
3^8	6561	1

The units digits appears to repeat every four powers of 3. To convince ourselves that this pattern continues, we notice that when starting with the integer 3 and continually multiplying by 3, there are a limited number of options for the resulting product's units digit. For any integer whose units digit is 3, when multiplied by 3, the resulting product's units digit is 9. For any integer whose units digit is 9, when multiplied by 3, the resulting product's units digit is 7. For any integer whose units digit is 7, when it is multiplied by 3, the resulting product's units digit is 1. For any integer whose units digit is 1, when it is multiplied by 3, the resulting product's units digit is 3. Thus, the cycle repeats.

Another way to convince ourselves that the pattern continues is by considering the fact that any positive integer can be written as $10a + b$, where a and b are

integers and $a \geq 0$ and $0 \leq b \leq 9$. Its product with 3 can be written as $3(10a + b) = 3(10a) + 3b$. Since $3(10a) = 3a(10)$, it must have a units digit of 0. Therefore, the units digit of $3(10a + b)$ must be the same as the units digit of $3b$. Thus, for any integer with units digit is 3 (so $b = 3$), when multiplied by 3, the resulting product's units digit is 9. For any integer whose units digit is 9 (so $b = 9$), when multiplied by 3, the resulting product's units digit is 7. For any integer whose units digit is 7 (so $b = 7$), when multiplied by 3, the resulting product's units digit is 1. For any integer whose units digit is 1 (so $b = 1$), when multiplied by 3, the resulting product's units digit is 3. Since the first power of 3 in our list, 3^1, has $b = 3$, we get the repeating cycle 3, 9, 7, and 1 of units digits.

Thus, we need to determine how many cycles of four there are in 2025 and if any remainder exists. We determine that $2025 \div 4 = 506$ R1. The remainder 1 corresponds to the first term of the cycle whose units digit is 3, meaning that the units digit of 3^{2025} is 3. Since the units digit is 3, the remainder when 3^{2025} is divided by 5 is 3.

Therefore, when 3^{2025} is divided by 5, the remainder is 3.

Problème de la semaine
Problème C
Déshydrater des fruits

La déshydratation permet de conserver les fruits en enlevant le surplus d'humidité.

La teneur en eau d'un certain fruit constitue 70% de sa masse. Donc, une autre matière que l'eau constitue 30% de sa masse.

Un fruit, lorsqu'il est laissé au soleil, perdra 80% de sa teneur en eau et la quantité d'autres matières reste la même.

Quel est le pourcentage d'eau, arrondi au dixième près, dans le fruit une fois déshydraté ?

THÈME : SENS DU NOMBRE
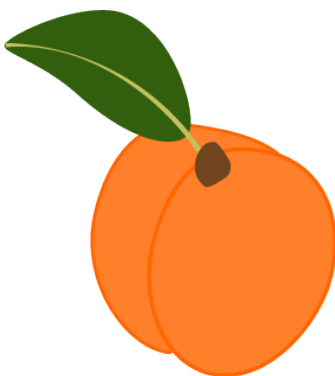
Problem of the Week
Problem C and Solution
Drying Fruit

Problem

Fruit can be preserved through drying to remove excess moisture.

The water content of a certain fruit, by mass, is 70%. Therefore, 30% of the fruit, by mass, is other material.

When left in the sun to dry, the fruit loses 80% of its water content, and the amount of other material remains the same.

Rounded to the nearest tenth, what percent of the dried fruit is water?

Solution

Solution 1

Let's consider a piece of fruit that originally weighs 100 g. Since 70% of the mass is water, that means that 70 g is water and 30 g is other material.

When left in the sun to dry, the fruit loses 80% of its water mass. So it loses 80% of 70 g = $0.8 \times 70 = 56$ g of water, and $70 - 56 = 14$ g of water remains.

The dried fruit still contains 30 g of other material. Therefore, the dried fruit consists of 14 g of water and 30 g other material, for a total of 44 g.

Therefore, the dried fruit is $\dfrac{14}{14 + 30} \times 100\% = \dfrac{14}{44} \times 100\% \approx 31.8\%$ water.

Solution 2

Suppose the fruit originally weighs x g. Since 70% of the mass is water, that means that 70% of $x = 0.7 \times x = 0.7x$ g is water and 30% of $x = 0.3x$ g is other material.

When left in the sun to dry, the fruit loses 80% of its water mass. So it loses 80% of $0.7x = 0.8 \times 0.7x = 0.56x$ g of water, and therefore $0.7x - 0.56x = 0.14x$ g of water remains.

The dried fruit still contains $0.3x$ g of other material. Therefore, the dried fruit consists of $0.14x$ g of water and $0.3x$ g of other material, for a total of $0.44x$ g.

Therefore, the dried fruit is $\dfrac{0.14x}{0.14x + 0.30x} \times 100\% = \dfrac{0.14x}{0.44x} \times 100\% \approx 31.8\%$ water.

Problème de la semaine
Problème C
La plus petite partie

Supposons que $N = 2^2 \times 3^2 \times 5^2 \times k$, où k est un nombre entier positif. Si N est divisible par 2025, quelle est la plus petite valeur possible de k ?

$$N = 2^2 \times 3^2 \times 5^2 \times k$$

THÈME : SENS DU NOMBRE

Problem of the Week
Problem C and Solution
The Smallest Part

Problem

Suppose $N = 2^2 \times 3^2 \times 5^2 \times k$, where k is a positive integer. If N is divisible by 2025, then what is the smallest possible value for k?

$$N = 2^2 \times 3^2 \times 5^2 \times k$$

Solution

First we note that $2025 = 3^4 \times 5^2$. Then, since N is divisible by 2025, N must have at least four factors of 3 and at least two factors of 5.

N already has two factors of 3 and two factors of 5. Thus, N needs at least two more factors of 3 in order to make it divisible by 2025. Therefore, the smallest possible value for k is $3^2 = 9$.

Problème de la semaine
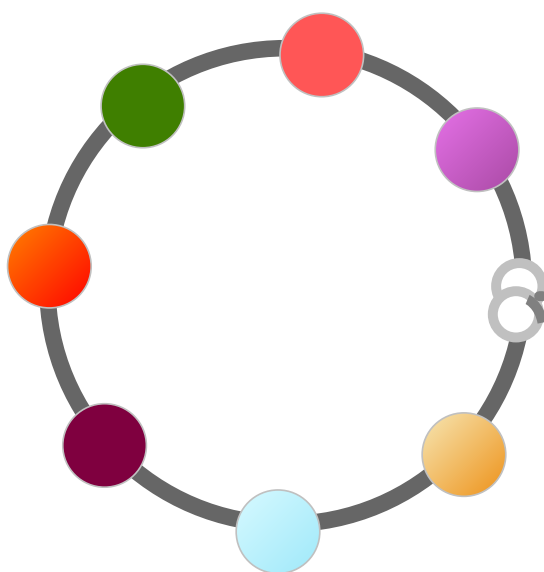## Problème C
Y aura-t-il quatre ou sept ?

Sophie s'occupe à créer deux styles de colliers : des petits colliers de quatre perles et de gros colliers de sept perles.

Elle a utilisé un total de 99 perles lorsqu'elle a fabriqué un certain nombre de petits et grands colliers. Détermine le nombre de possibilités pour chaque style de colliers que Sophie a créés.

THÈME : SENS DU NOMBRE

Problem of the Week
Problem C and Solution
Will it be Four or Seven?

Problem

Sophia has been making two types of necklaces: small necklaces that contain four beads each and large necklaces that contain seven beads each.

After creating a certain number of small and large necklaces, a total of 99 beads have been used. Determine all possibilities for how many of each type of necklace Sophia has made.

Solution

Let S represent the number of small necklaces and L represent the number of large necklaces she has made. Since S and L represent numbers of necklaces, both must be integers greater than or equal to 0. Since the small necklaces use 4 beads each, S necklaces would use $4 \times S$ or $4S$ beads in total. Since the large necklaces use 7 beads each, L necklaces would use $7 \times L$ or $7L$ beads in total. Since a total of 99 beads have been used, $4S + 7L = 99$.

We can also determine a maximum value for L. Since each large necklace uses 7 beads, $99 \div 7 \approx 14.1$, and L must be an integer, we know that L must be less than or equal to 14. Thus, L is an integer greater than or equal to 0 and less than or equal to 14. We could at this point check all of the possible integer values for L from 0 to 14. However, we can narrow down the possibilities even more.

In the equation, $4S + 7L = 99$, $4S$ will always be an even integer since 4 times any integer is always even. We have the even integer $4S$ plus $7L$ is equal to the odd integer 99. This means that $7L$ must be an odd integer. (The sum of an even integer and an even integer is an even integer, not an odd integer.) For $7L$ to be an odd integer, L must be odd. (If L is even, $7L$ would be even.) This observation reduces the possible values for L to the odd positive integers between 0 and 14, namely 1, 3, 5, 7, 9, 11, 13. For each possible value of L, we now determine $7L$, the total number of large beads used, $4S = 99 - 7L$, the total number of small beads used, and finally $S = (99 - 7L) \div 4$, the number of small necklaces for that value of L. If S is a non-negative integer, then we have found a valid possibility.

L	$7L$	$4S = 99 - 7L$	$S = (99-7L) \div 4$	Valid Possibility?
1	$7 \times 1 = 7$	$99 - 7 = 92$	$92 \div 4 = 23$	Yes, S is an integer
3	$7 \times 3 = 21$	$99 - 21 = 78$	$78 \div 4 = 19.5$	No, S is not an integer
5	$7 \times 5 = 35$	$99 - 35 = 64$	$64 \div 4 = 16$	Yes, S is an integer
7	$7 \times 7 = 49$	$99 - 49 = 50$	$50 \div 4 = 12.5$	No, S is not an integer
9	$7 \times 9 = 63$	$99 - 63 = 36$	$36 \div 4 = 9$	Yes, S is an integer
11	$7 \times 11 = 77$	$99 - 77 = 22$	$22 \div 4 = 5.5$	No, S is not an integer
13	$7 \times 13 = 91$	$99 - 91 = 8$	$8 \div 4 = 2$	Yes, S is an integer

Therefore, Sophia has either made 1 large necklace and 23 small necklaces, or 5 large necklaces and 16 small necklaces, or 9 large necklaces and 9 small necklaces, or 13 large necklaces and 2 small necklaces.

Problème de la semaine
Problème C
Dépêche-toi!

Ethan conduit sa trottinette électrique sur une distance de 6 km pour se rendre à sa pratique de basketball. En quittant la maison, il a regardé sa montre et a déterminé qu'il avait 24 minutes pour s'y rendre. Il a commencé son trajet à une vitesse de 12 km/h, mais après 12 minutes, il s'est rendu compte qu'il devait augmenter sa vitesse afin d'arriver à l'heure. À quelle vitesse doit-il voyager pour le reste du trajet afin que le voyage entier dure exactement 24 minutes ?

THÈME : SENS DU NOMBRE

Problem of the Week
Problem C and Solution
Hurry Up

Problem

Ethan is riding his e-scooter 6 km to basketball practice. When he left his house he checked his watch and determined that he had 24 minutes to get to practice. He started off at a speed of 12 km/hr, but after 12 min he realized he needs to increase his speed in order to get to practice on time. For the remaining time, what speed should Ethan ride at in order for the entire trip to take exactly 24 minutes?

Solution

The entire trip is 6 km and should take 24 minutes. Already Ethan has traveled for 12 min. That means the rest of his trip should take $24 - 12 = 12$ minutes.

For the first 12 minutes, Ethan rode at 12 km/h. This means that at that speed, in one hour, he would have traveled 12 km. Since 12 minutes is equal to $\frac{12}{60} = \frac{1}{5}$ of an hour, then the distance traveled during the first 12 minutes is $\frac{1}{5}$ of 12 km. This is $12 \div 5 = 2.4$ km. Since the total distance is 6 km, Ethan has to travel $6 - 2.4 = 3.6$ km in the remaining 12 min.

We need to determine the speed that Ethan should ride at to accomplish this. Ethan needs to travel 3.6 km in 12 min, or $\frac{1}{5}$ of an hour. At this speed, Ethan would travel $3.6 \times 5 = 18$ km in one hour. Therefore, Ethan must ride at 18 km/hr for the remaining time in order for the trip to take exactly 24 minutes.

Problème de la semaine
Problème C
Le nombre 1000, c'est toute une somme !

Savais-tu que le nombre 1000 peut s'écrire comme la somme de 16 nombres entiers consécutifs ?

Cela donne,

$$1000 = 55 + 56 + 57 + 58 + 59 + 60 + 61 + 62 + 63 + 64 + 65 + 66 + 67 + 68 + 69 + 70$$

Le *symbole de sommation Sigma*, utilisé dans l'équation ci-dessous, permet d'écrire de façon condensée la somme des nombres démontrée ci-haut.

$$\sum_{i=55}^{70} i = 1000$$

Quel est le nombre minimum de nombres entiers consécutifs que tu peux utiliser dans une équation dont la somme est de 1000 en utilisant au moins deux nombres entiers ?

THÈMES : SENS DU NOMBRE, ALGÈBRE

$$\sum_{i=55}^{70} i = 1000$$

Problem of the Week
Problem C and Solution
1000 is SUM Number!

Problem

Did you know that 1000 can be written as the sum of 16 consecutive integers?

That is,

$$1000 = 55 + 56 + 57 + 58 + 59 + 60 + 61 + 62 + 63 + 64 + 65 + 66 + 67 + 68 + 69 + 70$$

Using at least two integers, what is the minimum number of consecutive integers that sum to exactly 1000?

Solution

We will start with using two integers and then increase the number of integers by 1, until we discover the first number of integers that works.

- Can 1000 be written as the sum of two consecutive integers?

  Let n and $n + 1$ represent the two integers. Then we have

  $$n + (n + 1) = 1000$$
  $$2n + 1 = 1000$$
  $$2n = 999$$
  $$n = 499.5$$

  Since n is not an integer, it is not possible to write 1000 using two consecutive integers.

- Can 1000 be written as the sum of three consecutive integers?

  Let n, $n + 1$, $n + 2$ represent the three integers. Then we have

  $$n + (n + 1) + (n + 2) = 1000$$
  $$3n + 3 = 1000$$
  $$3n = 997$$
  $$n \approx 332.3$$

  Since n is not integer, it is not possible to write 1000 using three consecutive integers. (Refer to the note following the solution for an alternate way to represent three consecutive integers.)

- Can 1000 be written as the sum of four consecutive integers?

  Let n, $n + 1$, $n + 2$, $n + 3$ represent the four integers. Then we have

  $$n + (n + 1) + (n + 2) + (n + 3) = 1000$$
  $$4n + 6 = 1000$$
  $$4n = 994$$
  $$n = 248.5$$

  Since n is not an integer, it is not possible to write 1000 using four consecutive integers.

- Can 1000 be written as the sum of five consecutive integers?

  Let n, $n+1$, $n+2$, $n+3$, $n+4$ represent the five integers. Then we have

  $$n + (n+1) + (n+2) + (n+3) + (n+4) = 1000$$
  $$5n + 10 = 1000$$
  $$5n = 990$$
  $$n = 198$$

  Since n is an integer, it is possible to write 1000 using five consecutive integers. That is, as $1000 = 198 + 199 + 200 + 201 + 202$.

Therefore, the minimum number of consecutive integers that sum to 1000 is five.

NOTE:

In the second case, when we checked if 1000 could be written as the sum of three consecutive integers, we could have proceeded as follows:

Let $a - 1$, a, $a + 1$ represent the three consecutive integers. Then we have

$$(a - 1) + a + (a + 1) = 1000$$
$$3a = 1000$$
$$a \approx 333.3$$

Since a is not an integer, it is not possible to write 1000 using three consecutive integers.

This idea is useful when we are finding the sum of an odd number of consecutive integers. If we applied the same idea to the fourth case by using $a - 2$, $a - 1$, a, $a + 1$, $a + 2$ to represent the five consecutive integers, we would have

$$(a - 2) + (a - 1) + a + (a + 1) + (a + 2) = 1000$$
$$5a = 1000$$
$$a = 200$$

Thus, $a - 2 = 198$, $a - 1 = 199$, $a + 1 = 201$, and $a + 2 = 202$. And so it is possible to write 1000 using five consecutive integers as $1000 = 198 + 199 + 200 + 201 + 202$.

FOR FURTHER THOUGHT:

What is the largest **odd** number of consecutive positive integers that can be used to sum to 1000?

How would your answer change if the word positive was removed from the above sentence?

Problème de la semaine
Problème C
Carrelage efficace

Sophia a une réserve illimitée de carreaux carrés. Elle possède des carreaux de 1 cm sur 1 cm, 2 cm sur 2 cm, 3 cm sur 3 cm, et ainsi de suite. Chaque carreau a une longueur de côté exprimée en un nombre entier de centimètres.

Un plancher de salle de bain rectangulaire, d'une aire de 84 cm sur 140 cm, doit être recouvert de carreaux carrés de dimension identique, sans qu'on doive les couper. Sophia sait que le plancher peut être recouvert par 11 760 carreaux de 1 cm sur 1 cm, puisque $84 \times 140 = 11\,760$ cm^2. Cependant, elle souhaite utiliser le plus petit nombre possible de carreaux de même taille pour accomplir cette tâche.

Détermine le nombre minimal de carreaux de taille identique nécessaires pour recouvrir complètement le plancher de la salle de bain.

THÈMES : GÉOMÉTRIE ET MESURE, SENS DU NOMBRE

Problem of the Week
Problem C and Solution
Efficient Tiling

Problem

Sophia has an unlimited supply of square tiles. Sophia has 1 cm by 1 cm tiles, 2 cm by 2 cm tiles, 3 cm by 3 cm tiles, and so on. Every tile has integer side lengths, in centimetres.

A rectangular bathroom floor with an 84 cm by 140 cm surface is to be completely covered by identical square tiles, none of which can be cut. Sophia knows that the floor can be completely covered with 11 760 tiles of size 1 cm by 1 cm, since $84 \times 140 = 11\,760$ cm^2. However, Sophia wants to use the minimum number of identical sized tiles to complete the job.

Determine the minimum number of identical sized tiles required to completely cover the bathroom floor.

Solution

To use the least number of tiles, Sophia must use the largest tile possible. The square tile must have sides less than or equal to 84 cm. If the tile has side length greater than 84 cm, it would have to be cut to fit the width of the floor.

Since the tiles are square and must completely cover the floor, the side length of the tile must be a number that is a factor of both 84 and 140. In fact, since Sophia wants the largest side length, she is looking for the greatest common factor of 84 and 140.

The positive factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, and 84.

The positive factors of 140 are 1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70, and 140.

The largest number common to both lists is 28. Therefore, the greatest common factor of 84 and 140 is 28. The required tiles are 28 cm by 28 cm. Since $84 \div 28 = 3$, the surface is 3 tiles wide. Since $140 \div 28 = 5$, the surface is 5 tiles long. The minimum number of tiles required is $3 \times 5 = 15$ tiles.

The number of 28 cm by 28 cm tiles required to cover the floor area is 15. This is the minimum number of tiles required.

Problème de la semaine
Problème C
Partage équitable

Piero a 7 tuiles, chacune portant un nombre entier différent de 1 à 7. Il existe plusieurs façons de regrouper ses tuiles en formant des ensembles contenant chacun au moins une tuile. Par exemple, il peut diviser ses tuiles en 3 groupes, comme illustré.

$$\boxed{1}\boxed{2}\boxed{4}\boxed{6} \qquad \boxed{3}\boxed{7} \qquad \boxed{5}$$

La somme des nombres dans chacun de ces groupes est respectivement 13, 10 et 5.

Piero décide ensuite de former différents groupes avec ses tuiles et remarque que, cette fois, la somme des nombres dans chaque groupe est identique. De combien de façons différentes peut-il répartir ses tuiles en au moins deux groupes, de sorte que la somme des nombres dans chaque groupe soit la même ?

Problem of the Week
Problem C and Solution
Group Work

Problem

Piero has 7 tiles, each with a different integer from 1 to 7 written on it. There are many ways in which he can separate his tiles into groups, where each group contains at least one tile. For example, he can separate his tiles into 3 groups as shown.

$$\boxed{1}\;\boxed{2}\;\boxed{4}\;\boxed{6} \qquad\qquad \boxed{3}\;\boxed{7} \qquad\qquad \boxed{5}$$

The sum of the numbers in each of these groups is 13, 10, and 5, respectively.

Piero then separates his tiles into different groups and notices that the sum of the numbers in each group is the same. In how many different ways can Piero separate his tiles into at least two groups so that the sum of the numbers in each group is the same?

Solution

The sum of the numbers on Piero's tiles is $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. If the sum of the numbers in each group is the same, then this sum must be a factor of 28. The positive factors of 28 are 1, 2, 4, 7, 14, and 28. Since the number 7 must be in one of the groups, it follows that the sum of each group must be at least 7. Also since there must be at least 2 groups, then the sum of each group cannot be 28, because in that case there would be only 1 group. It follows that the groups could each have a sum of 7 or a sum of 14. We will look at these two cases.

Case 1: Each group has a sum of 7.
In this case, there would be $28 \div 7 = 4$ groups. The only way to separate the tiles into 4 groups, each with a sum of 7, is as $\{7\}$, $\{1, 6\}$, $\{2, 5\}$, $\{3, 4\}$.

Case 2: Each group has a sum of 14.
In this case, there would be $28 \div 14 = 2$ groups. To help count the possible groups, we note that one of the two groups must contain the tile with the 7. The possibilities for the other tiles are as follows:

- If the group with the 7 also contains the 6, then the only possibility is $\{1, 6, 7\}$. The other group must then be $\{2, 3, 4, 5\}$.

- If the group with the 7 also contains the 5, then the only possibility is $\{2, 5, 7\}$. The other group must then be $\{1, 3, 4, 6\}$.

- If the group with the 7 also contains the 4, then the group could be either $\{3, 4, 7\}$ or $\{1, 2, 4, 7\}$. The other group would then be $\{1, 2, 5, 6\}$ or $\{3, 5, 6\}$, respectively.

- If the group with the 7 also contains the 3, 2, or 1, then it must also contain the 4, 5, or 6, since $7 + 3 + 2 + 1 = 13$, which is less than 14. So there are no additional cases.

Thus, there are 4 ways to separate the tiles into 2 groups, each with a sum of 14.

Therefore, there are $1 + 4 = 5$ different ways in which Piero can separate his tiles into at least two groups so that the sum of the numbers in each group is the same. These are listed below.

- $\{7\}$, $\{1, 6\}$, $\{2, 5\}$, $\{3, 4\}$

- $\{1, 6, 7\}$, $\{2, 3, 4, 5\}$

- $\{2, 5, 7\}$, $\{1, 3, 4, 6\}$

- $\{3, 4, 7\}$, $\{1, 2, 5, 6\}$
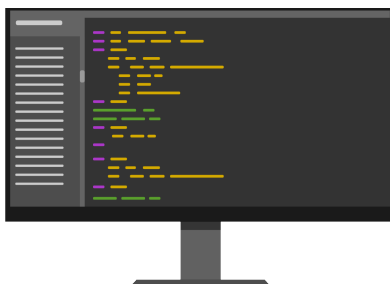
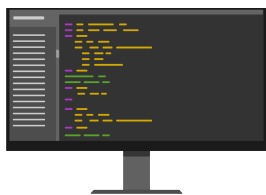- $\{1, 2, 4, 7\}$, $\{3, 5, 6\}$

Problème de la semaine
Problème C
Programme de sommes

Ruben a écrit un programme qui utilise une liste de nombres comme données d'entrée. Le programme additionne ensuite chaque paire de nombres adjacents de la liste pour obtenir une nouvelle liste. Ensuite, il répète ce processus avec la nouvelle liste jusqu'à ce qu'il ne reste plus qu'un seul nombre. Ce nombre constitue alors la sortie.

Par exemple, si l'entrée est $(1,\ 5,\ 3,\ 2)$, le programme additionne les nombres adjacents pour obtenir $(6,\ 8,\ 5)$. Il répète ensuite le processus pour obtenir $(14,\ 13)$, puis (27). Comme la liste ne contient plus qu'un seul nombre, la sortie est 27.

Soit R le nombre préféré de Ruben. Ruben a donc entré les nombres $(6,\ 4,\ R,\ 7)$ dans son programme et la sortie était 4 fois son nombre préféré, soit $4R$. Quel est alors le nombre préféré de Ruben ?

THÈMES : ALGÈBRE, SENS DU NOMBRE

Problem of the Week
Problem C and Solution
Sum Program

Problem

Ruben wrote a program that takes a list of numbers as input. The program then adds each pair of adjacent numbers in the list to obtain a new list of numbers, then repeats this process with the new list until the list contains only one number. This number is then the output.

For example, if the input is $(1, 5, 3, 2)$ then the program adds the adjacent numbers in the list to obtain $(6, 8, 5)$. The program then repeats this process to obtain $(14, 13)$, and then (27). Since the list now contains only one number, the output is 27.

Let R represent Ruben's favourite number. Ruben input the numbers $(6, 4, R, 7)$ into his program and the output was 4 times his favourite number, or $4R$. What is Ruben's favourite number?

Solution

We could attempt this problem using trial and error, and perhaps we would even stumble on the correct answer. However, this may take a long time and is not an efficient method for solving the problem. Instead we will provide a more algebraic solution.

The list of numbers inputted into the program was $(6, 4, R, 7)$. After the adjacent numbers are added together, we obtain $(10, 4 + R, R + 7)$. After the adjacent numbers in this list are added together, we obtain $(10 + 4 + R, 4 + R + R + 7)$, which is equal to $(14 + R, 11 + 2R)$. After the adjacent numbers in this list are added together, we obtain $(14 + R + 11 + 2R)$ which is equal to $(25 + 3R)$. Since there is now one number in the list, this number should be equal to the output of the program, which is $4R$. Thus,

$$25 + 3R = 4R$$
$$25 + 3R - 3R = 4R - 3R$$
$$25 = R$$

Therefore, Ruben's favourite number is 25.