



Problem of the Week

Problem B and Solution

Fun and Games

Problem

Mariana's trivia team is playing a game against other teams. The game includes 10 questions, and each team is given 5 points to start. Teams then earn 4 points for each correct answer, and lose 3 points for each incorrect answer.

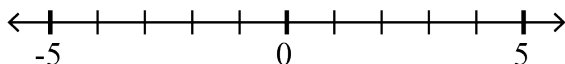


- (a) The table shows the score card for Mariana's team.

(The trivia questions were tough!)

In the right column, fill in their *running score*, which is their current score after each question.

Note that their running score may be positive or negative, like a temperature reading. You may find the following number line helpful.



Question	Result	Running Score
—	—	5
1	X	
2	X	
3	X	
4	✓	
5	X	
6	✓	
7	X	
8	✓	
9	X	
10	✓	

- (b) The winning team had a score of 10. How many questions did they get correct?

EXTENSION: What is the maximum possible score? What is the minimum possible score? Are all the integers in between these values possible scores? Explain.



Solution

(a) The running score is shown in the following table.

Question	Result	Running Score
—	—	5
1	X	$5 - 3 = 2$
2	X	$2 - 3 = -1$
3	X	$-1 - 3 = -4$
4	✓	$-4 + 4 = 0$
5	X	$0 - 3 = -3$
6	✓	$-3 + 4 = 1$
7	X	$1 - 3 = -2$
8	✓	$-2 + 4 = 2$
9	X	$2 - 3 = -1$
10	✓	$-1 + 4 = 3$

(b) The winning team's score went from 5 to 10, so we know they gained 5 points. Notice that the total score for a correct answer followed by an incorrect answer is $4 - 3 = 1$. So 5 pairs of correct and incorrect answers would have a total score of 5. This would also be 10 questions in total, as desired. Therefore, the winning team must have had 5 questions correct.

SOLUTION TO EXTENSION:

If a team got every question correct, they would earn $10 \times 4 = 40$ points, plus the 5 points at the beginning, for a total of $40 + 5 = 45$ points. This is the maximum possible score.

If a team got every question incorrect, they would lose $10 \times 3 = 30$ points, but since they started off with 5 points, their score would be $5 - 30 = -25$ points. This is the minimum possible score.

All the integers in between the minimum and maximum are **not** possible scores. The number of correct answers can be any integer from 0 to 10. Thus, there are 11 possibilities for the number of correct answers. Therefore, there cannot be more than 11 different possible scores. However, there are more than 11 integers between -25 and 45 so not all of them can be possible scores.

Another way to show this is to determine all the 11 possible scores. They are $-25, -18, -11, -4, 3, 10, 17, 24, 31, 38$, and 45 .