Problem of the Week
Problem B and Solution
Loose Change

Problem

Macklin has ten coins in his pocket worth a total of \$2.30. Only one of Macklin's coins is a nickel (worth \$0.05), and the other nine are some combination of dimes (worth \$0.10), quarters (worth \$0.25), and loonies (worth \$1.00), with at least one of each type of coin.

(a) How many dimes does Macklin have?

(b) Is it possible that Macklin could have two nickels, not one? Explain your reasoning.

Solution

(a) We know that Macklin has at least one of each type of coin, so we will start by taking one of each type of coin and recording their total value in the following table.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	1	1	1	1	4
Value (\$)	0.05	0.10	0.25	1.00	1.40

This gives a total value of \$1.40, so we have $\$2.30 - \$1.40 = \$0.90$ remaining. Since \$0.90 is less than the value of a loonie, Macklin cannot have any more loonies. Also, since only one of Macklin's coins is a nickel, Macklin cannot have any more nickels. Thus, we need to find a way to make \$0.90 using only quarters and/or dimes. The only way to do so is by using no quarters and 9 dimes, or by using 2 quarters and 4 dimes, since
$(2 \times \$0.25) + (4 \times \$0.10) = \$0.90$.

Using 9 dimes uses a total of $4 + 9 = 13$ coins. Since Macklin has 10 coins, this will not work.

Using 2 quarters and 4 dimes uses a total of $4 + 2 + 4 = 10$ coins, as desired. We will add these coins to our table.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	1	5	3	1	10
Value (\$)	0.05	0.50	0.75	1.00	2.30

Thus, Macklin has 5 dimes in total.

(b) If two of Macklin's coins were nickels, then we can proceed like in part (a), but include an extra nickel.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	2	1	1	1	5
Value ($)	0.10	0.10	0.25	1.00	1.45

This gives a total value of \$1.45, so we have $\$2.30 - \$1.45 = \$0.85$ remaining. As before, we can make this amount using only quarters and/or dimes.

Using 3 quarters and 1 dime we can make \$0.85, however this uses a total of $5 + 3 + 1 = 9$ coins, which is not enough.

Using 1 quarter and 6 dimes we can make \$0.85, however this uses a total of $5 + 1 + 6 = 12$ coins, which is too many.

There are no other ways to make \$0.85 using only quarters and/or dimes. Thus, Macklin could not have had two nickels.