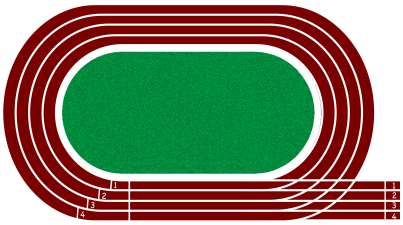
Problem of the Week
Problem B and Solution
Who Wins the Best Lane?

Problem

Donovan Bailey Elementary School has organized a relay race as part of its track and field day. Four teams, A, B, C, and D, are participating.

To assign each team to a lane on the track, the principal draws the team names randomly. The first team drawn will run in Lane 1, the next in Lane 3, and the remaining team will run in Lane 4.



(a) What is the theoretical probability that Team A will be assigned to Lane 1? Express your answer as a fraction, a decimal, and a percentage.

(b) The principal decides to draw teams several times, and assign Lane 1 to the team that was drawn first most often. The results are in the table.

Team	A	B	C	D
Number of Times First	20	6	12	10

Use these results to calculate the experimental probability of Team A being assigned to Lane 1. How does this compare to the theoretical probability from part (a)?

(c) Use the theoretical probability from part (a) to calculate the number of times you would expect each team to win Lane 1. Do any of the experimental results in part (b) equal this expected result?

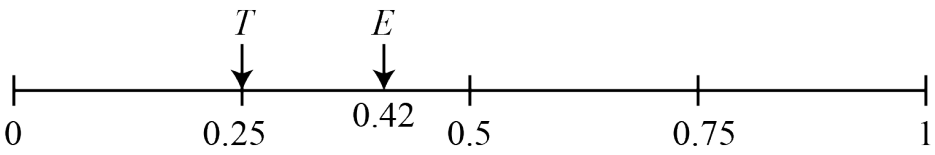
Extension:

Draw a spinner with four sections so that the theoretical probability of landing on each section is the same as the experimental probability calculated from the table.

Solution

(a) Since there are 4 teams, and each team has an equal chance of being assigned to Lane 1, the theoretical probability that Team A will be assigned to Lane 1 is $\frac{1}{4}$, or 0.25, or 25%.

(b) The principal drew teams $20 + 6 + 12 + 10 = 48$ times. Thus, the experimental probability of Team A being assigned to Lane 1 was $\frac{20}{48}$, or $\frac{5}{12}$, which is approximately 0.42, or 42%. This is significantly higher than the theoretical probability from part (a). They are shown on the following probability number line, where E represents the experimental probability and T represents the theoretical probability.



(c) Since the principal drew teams 48 times, and each team had an equal chance of getting drawn each time, we would expect that the number of times each team would win Lane 1 would be $48 \div 4 = 12$. The experimental result for Team C is equal to this expected result.

Solution to Extension:

When creating the spinner, we note that the probability of landing on section C should be $\frac{1}{4}$. Since a circle has 360°, this means that the centre angle for section C should be $\frac{1}{4}$ of 360°, which is 90°.

We notice that the number of times first for Team B is half the number of times first for Team C. Thus, the centre angle for section B should be half of 90°, which is 45°. This leaves a section with a centre angle of $360° - 90° - 45° = 225°$ that will be split between sections A and D.

We notice that the number of times first for Team A is twice the number of times first for Team D. Thus, section A should be twice as large as section D. Since they're sharing a section with a centre angle of 225°, we can determine that $\frac{2}{3}$ of this section will be section A and $\frac{1}{3}$ will be section D. Therefore, the centre angle for section A will be $\frac{2}{3}$ of 225°, which is 150°, and the centre angle for section D will be $\frac{1}{3}$ of 225°, which is 75°. Now we use our protractor to draw the spinner as shown.