



Problem of the Week

Problem B and Solution

Grain Guess

Problem

Caden's class wants to win a prize for the closest estimate to the number of grains of rice in a 1 kg bag of rice.

They have been given three clues:

Clue (1): The number of grains is a 5-digit even number.

Clue (2): The sum of the five digits in the number is 15.

Clue (3): The number, rounded to the nearest hundred, is 51 200.

- (a) Help Caden's class win the prize. Use the given clues to determine all possible estimates of the number of grains of rice in the bag.
- (b) Kendala, one of Caden's classmates, found that the mass of a single grain of rice depends on the type, ranging from about 20 to 45 milligrams. When she used this to calculate how many grains were in the 1 kg bag, did her answer suggest the rice was of the lighter or heavier type?

Solution

- (a) From Clues (1) and (3), we know that the number is an even number between 51 150 and 51 249.

Therefore, the number is of the form 51 1 XY or 51 2 XY , where X and Y are digits.

Suppose the number is of the form 51 1 XY . We know that Y is even, and $X \geq 5$. Since $5 + 1 + 1 = 7$, then from Clue (2), $X + Y = 8$. Therefore, we could have $X = 6$ and $Y = 2$, or $X = 8$ and $Y = 0$.

Suppose the number is of the form 51 2 XY . We know that Y is even and $X \leq 4$. Since $5 + 1 + 2 = 8$, then from Clue (2), $X + Y = 7$. Since Y is an even digit, we could have $X = 3$ and $Y = 4$, or $X = 1$ and $Y = 6$.

Therefore, the estimates that satisfy all three clues are 51 162, 51 180, 51 234, and 51 216.

- (b) Since $1 \text{ kg} = 1000 \text{ g} = 1\,000\,000 \text{ mg}$, for the lighter rice, Kendala would calculate $1\,000\,000 \div 20 = 50\,000$ grains of rice. For the heavier rice, she would calculate $1\,000\,000 \div 45 \approx 22\,222$ grains of rice. Thus, her answers indicate that the class's rice was of the lighter type, since that answer is closest to their estimate.