Problem of the Week
Problem B and Solution
Smoothie Choices

Problem

Justin and his siblings make smoothies every morning. Their smoothie recipe uses 3 cups of fruit. Sometimes they use only one type of fruit and sometimes they use more than one type of fruit, but they always use exactly 1, 2, or 3 cups of each fruit.

Smoothies taste different when they change the amounts of each fruit. For example, a smoothie with 2 cups of mango and 1 cup of blueberries is different than a smoothie with 1 cup of mango and 2 cups of blueberries.

(a) If they have only strawberries and bananas, how many different smoothies can they make?

(b) If they have strawberries, bananas, and raspberries, how many different smoothies can they make?

(c) If they have strawberries, bananas, raspberries, and peaches, how many different smoothies can they make?

Solution

(a) To determine the number of smoothies they can make using only bananas and/or strawberries, we will write out all the options. Let S represent 1 cup of strawberries and B represent 1 cup of bananas. The smoothies they can make are as follows:

$$SSS,\ SSB,\ SBB,\ BBB$$

Thus, they can make 4 different smoothies if they have only bananas and strawberries.

(b) Let R represent 1 cup of raspberries. The smoothies they can make using strawberries, bananas, and/or raspberries are as follows:

$$SSS,\ BBB,\ RRR,\ SSB,\ SBB,\ SSR,\ SRR,\ BBR,\ BRR,\ SBR$$

Thus, they can make 10 different smoothies if they have strawberries, bananas, and raspberries.

(c) Let P represent 1 cup of peaches. Since there's more to count, we'll organize the smoothies by the number of cups of peaches.

- **Smoothies with 0 cups of peaches**: From part (b), we know that they can make 10 different smoothies if they have strawberries, bananas, and raspberries.
- **Smoothies with 1 cup of peaches**: The possibilities for the remaining 2 cups of fruit are SS, BB, RR, SB, SR, and BR. Therefore, they can make 6 different smoothies if 1 cup is peaches.
- **Smoothies with 2 cups of peaches**: The possibilities for the remaining 1 cup of fruit are S, B, and R. Therefore, they can make 3 different smoothies if 2 cups are peaches.
- **Smoothies with 3 cups of peaches**: This can be done in only one way as PPP.

Thus, in total they can make $10 + 6 + 3 + 1 = 20$ different smoothies if they have strawberries, bananas, raspberries, and peaches.