



## Problem of the Week

### Problem B and Solution

### They All Add Up

#### Problem

Etta is finding the sum of the digits of numbers. For example, the sum of the digits in 904 is  $9 + 0 + 4 = 13$ .

- (a) Etta determines that there are 15 integers from 1 to 1000 whose digits have a sum of 4. Find all these integers.
- (b) What fraction of these integers are even?

#### Solution

- (a) First we look at the integers less than 10. The only integer less than 10 whose digits sum to 4 is the number 4 itself.

Next we look at integers between 10 and 99. If the two digits in the integer add to 4, then the digits could be 0 and 4, 1 and 3, or 2 and 2. These pairs of digits and the possible integers they create are summarized in the following table.

Pairs of Digits	Possible Integers
0, 4	40
1, 3	13, 31
2, 2	22

Finally we look at the integers between 100 and 999. Since the digits in 1000 don't have a sum of 4, we can consider only the three-digit numbers. The groups of digits that add to 4 and the possible integers they create are summarized in the following table.

Groups of Digits	Possible Integers
0, 0, 4	400
0, 1, 3	103, 130, 301, 310
0, 2, 2	220, 202
1, 1, 2	112, 121, 211

Therefore the 15 integers whose digits have a sum of 4 are:

4, 13, 22, 31, 40, 103, 112, 121, 130, 202, 211, 220, 301, 310, 400

- (b) Of these 15 integers, 9 are even. So the fraction of the integers that are even is  $\frac{9}{15}$ , or  $\frac{3}{5}$ .