Problem of the Week
Problem B and Solution
Create a Magic Square

Problem

In a *magic square*, the sum of the numbers in each row, column, and diagonal is the same. This number is called the *magic sum*. In the following magic square, A, B, C, D, E, F, G, H, and J represent numbers, and clues are given to determine some of these numbers.

- The value of A is one more than the product of 3 and 4.

- The value of B is the sum of 1, 2, and 3.

- The value of C is the smallest odd number greater than 10.

- The value of F is also known as a dozen.

A	B	C
D	E	F
G	H	J

- The value of G is equal to the total number of sides in three triangles.

(a) Use the given clues to determine the values for A, B, C, F, and G.

(b) Determine the magic sum for this magic square, then use this to complete the magic square.

(c) Would you have been able to complete this magic square if only four clues were given, instead of five? If so, which clue(s) do you not need?

Solution

(a) The product of 3 and 4 is $3 \times 4 = 12$. One more than that is $12 + 1 = 13$. Therefore, $A = 13$.

The sum of 1, 2, and 3 is $1 + 2 + 3 = 6$. Therefore, $B = 6$.

The smallest odd number greater than 10 is 11. Therefore, $C = 11$.

A dozen is 12. Therefore, $F = 12$.

Since one triangle has 3 sides, then the total number of sides in three triangles is $3 \times 3 = 9$. Therefore, $G = 9$.

(b) The magic sum is equal to $A + B + C = 13 + 6 + 11 = 30$. Then, since we know A and G, it follows that $D = 30 - A - G = 30 - 13 - 9 = 8$. Since we know C and G, then $E = 30 - C - G = 30 - 11 - 9 = 10$. Since we know B and E, then $H = 30 - B - E = 30 - 6 - 10 = 14$. Finally, since we know C and F, then $J = 30 - C - F = 30 - 11 - 12 = 7$. The completed magic square is shown.

13	6	11
8	10	12
9	14	7

(c) Of the five given clues, the first three clues about A, B, and C are needed to determine the magic sum. Since we need the magic sum to complete the magic square, it follows that we need these three clues.

In our solution for (b), before the final step, we could have determined F using D and E. So it's possible to complete the magic square given only the clues about A, B, C, and G.

Alternatively, we could have first used C and F to determine J. Then we could have used A and J to determine E, and then used E and F to determine D. Then we could have used A and D to determine G, and then used B and E to determine H. So it's also possible to complete the magic square given only the clues about A, B, C, and F. Therefore, we could have completed the magic square without the clue for F or without the clue for G.