



Problem of the Week

Problem B and Solution

Does This All Add Up?

Problem

In the table shown, the top row and leftmost column are grey, and the remaining numbers form a 3×3 array. Each number in the array is equal to the sum of the numbers in grey in its row and column.

	5	2	7
3	8	5	10
4	9	6	11
6	11	8	13

(a) Follow the steps below.

1. Circle any number in the array (*for example, 6*).
2. Cross off the other numbers in the same row and column of the array.
(*For our example, we would then cross out 9, 11, 5, and 8.*)
3. Circle any remaining number in the array (*for example, 13*).
4. Cross off the other numbers in the same row and column of the array.
(*For our example, we would then cross out 10 and 11.*)
5. Circle the remaining number in the array (*for our example, this is 8*).

What is the sum of the three circled numbers?

(b) Repeat the steps in part (a) two more times, starting with a different number each time. What do you notice about the sum of the three circled numbers?

(c) Will your result from (b) be true if we create a 3×3 array using different initial numbers in the grey row and column? Explain why it will be true or give an example where it would not be true.



Solution

- (a) The sum of the three circled numbers in our example is $6 + 13 + 8 = 27$.
- (b) The sum of the three circled numbers is always 27. Notice that this is also the sum of the six numbers in the grey row and column:
 $5 + 2 + 7 + 3 + 4 + 6 = 27$.
- (c) Yes, the result will always be true. Regardless of which number is circled first, after the steps are completed there will be exactly one circled number in each row and column. This is because when a number is circled, the other numbers in its row and column are crossed out. Since each number in the array is the sum of the grey numbers in its row and column, it follows that the sum of the three circled numbers will always be equal to the sum of the six grey numbers.

EXTENSION: If each number in the array was instead the *product* of the grey numbers in its row and column, would the product of the three circled numbers equal the product of the six grey numbers?