Problem of the Week

Problems and Solutions 2024-2025

Problem B

Grade 5/6

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

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Algebra (A)

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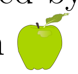
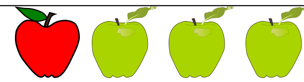
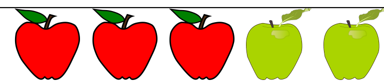
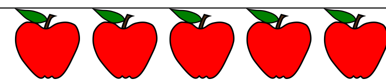
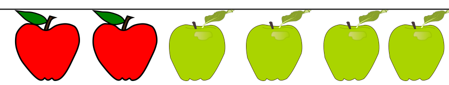
Problem of the Week

Problem B

The Apple of Your Eye

Four friends go to a local apple orchard to pick apples. The table below summarizes how many of baskets of each variety were picked by each friend. Each  represents a basket of Macintosh apples and each  represents a basket of Golden Delicious apples.

Student	**Apples Picked**
Artur	
Khalil	
Zendaya	
Georgina	

(a) Let m be the number of apples in a basket of Macintosh apples and g be the number of apples in a basket of Golden Delicious apples. For each friend, create an algebraic expression in terms of m and g for the total number of apples they picked.

(b) Let T be the total number of apples that all of the friends picked. Combine like terms to create an algebraic equation for T in terms of m and g.

(c) Suppose there are 12 apples in each basket of Macintosh apples and 10 apples in each basket of Golden Delicious apples. What is the total number of apples picked by the friends?

Theme: Algebra

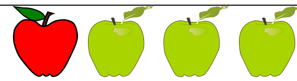
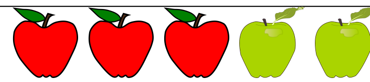
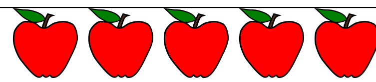
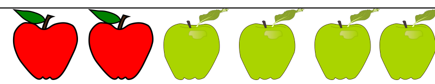
Problem of the Week

Problem B and Solution

The Apple of Your Eye

Problem

Four friends go to a local apple orchard to pick apples. The table below summarizes how many of baskets of each variety were picked by each friend. Each  represents a basket of Macintosh apples and each  represents a basket of Golden Delicious apples.

Student	Apples Picked
Artur	
Khalil	
Zendaya	
Georgina	

(a) Let m be the number of apples in a basket of Macintosh apples and g be the number of apples in a basket of Golden Delicious apples. For each friend, create an algebraic expression in terms of m and g for the total number of apples they picked.

(b) Let T be the total number of apples that all of the friends picked. Combine like terms to create an algebraic equation for T in terms of m and g.

(c) Suppose there are 12 apples in each basket of Macintosh apples and 10 apples in each basket of Golden Delicious apples. What is the total number of apples picked by the friends?

Solution

(a) We have the following algebraic expressions for the number of apples picked by each friend:

- Artur: $1m + 3g$
- Khalil: $3m + 2g$
- Zendaya: $5m$
- Georgina: $2m + 4g$

(b) Using the expressions in part (a), we have
$T = m + 3m + 5m + 2m + 3g + 2g + 4g = 11m + 9g$.

(c) The total number of the Macintosh apples is $11 \times 12 = 132$ apples. The total number of the Golden Delicious apples is $9 \times 10 = 90$ apples.
Therefore, the total number of apples they picked is $132 + 90 = 222$ apples.

An alternate way to determine the number of apples is to substitute $m = 12$ and $g = 10$ into the equation $T = 11m + 9g$ and follow the order of operations to get
$T = 11 \times 12 + 9 \times 10 = 132 + 90 = 222$ apples.

Problem of the Week

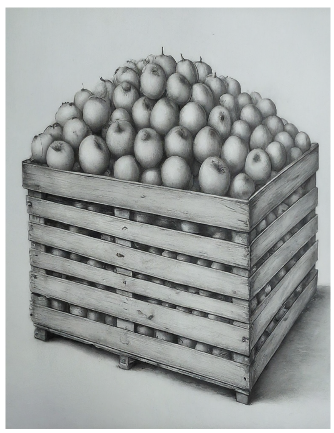
Problem B

Picking Fruit

Kelvin has a part-time job picking apples. Each day, he starts with an empty container, which can hold 960 apples, and puts the apples that he picks into the container. Kelvin is finished his work for the day if either 3 hours have passed or he has filled the container.

Kelvin picks 400 apples on the first day, and each day steadily improves the rate at which he picks apples by picking 21 more apples than the day before.

After how many days of working can he fill the container in less than 3 hours?



Theme: Algebra

Problem of the Week
Problem B and Solution
Picking Fruit

Problem

Kelvin has a part-time job picking apples. Each day, he starts with an empty container, which can hold 960 apples, and puts the apples that he picks into the container. Kelvin is finished his work for the day if either 3 hours have passed or he has filled the container.

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After how many days of working can he fill the container in less than 3 hours?

Solution

Kelvin picks 400 apples the first day, 421 apples the second day, 442 apples the third day, and so on, increasing each day by 21 apples. To find the first day he can completely fill the container in less than 3 hours, we need to determine the number of days it takes for him to increase his productivity by $960 - 400 = 560$ apples.

Since, $560 \div 21 \approx 26.6$, there are 26 days of *increase* in apple picking, but where Kelvin picks an amount that is still less than 960 apples. That is, on day 1 through day 27, Kelvin picks less than 960 apples. Then on day 28, Kelvin picks 960 or more extra apples. That is, on day 28 and thereafter, he can fill the container in less than 3 hours.

Indeed, we can check that on day 27 Kelvin picks $21 \times 26 = 546$ extra apples, for a total of $400 + 546 = 946$ apples. On day 28, Kelvin picks $21 \times 27 = 567$ extra apples, for a total of $400 + 567 = 967$ apples in 3 hours.

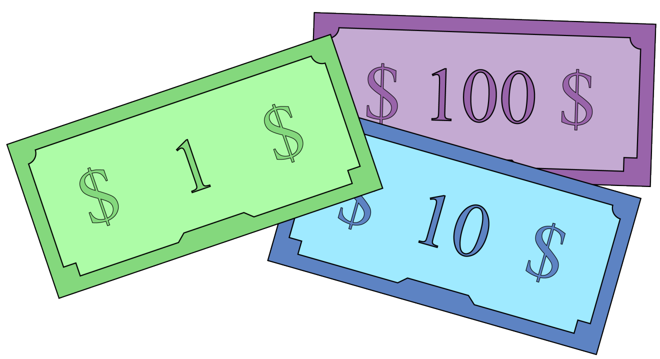
Problem of the Week
Problem B
Ask the Banker

Vesna is the banker in a board game that uses \$1, \$10, and \$100 bills.

(a) Vesna needs to give a player \$2163. How can you do this using the fewest total number of bills? How can you do this using the greatest total number of bills?

(b) Is it possible to give a player \$254 using exactly 20 bills in total? How about using exactly 30 bills in total? If so, show how it's possible. If not, explain why it's not possible.

Extension: Vesna likes when she can give a player the same number of each type of bill. For which total amounts of money is this possible? Explain.



Themes: Algebra, Number Sense

Problem of the Week
Problem B and Solution
Ask the Banker

Problem

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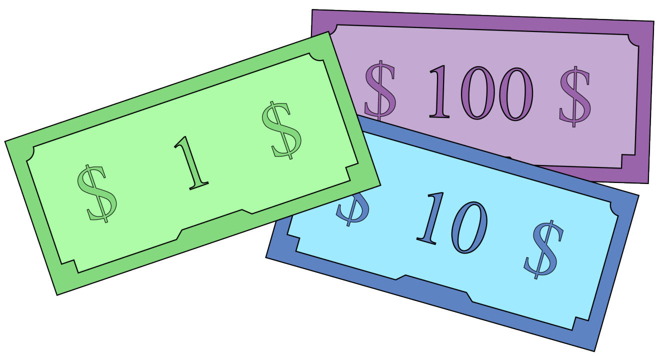
EXTENSION: Vesna likes when she can give a player the same number of each type of bill. For which total amounts of money is this possible? Explain.



Solution

(a) To give a player \$2163 using the fewest total number of bills, we first use as many \$100 bills as we can. Since the number 2163 has 21 hundreds, then we can use at most 21 of the \$100 bills. This gives $21 \times \$100 = \$2100$, so we are left with $\$2163 - \$2100 = \$63$. Next we use as many \$10 bills as we can. Since the number 63 has 6 tens, then we can use at most 6 of the \$10 bills. This gives $6 \times \$10 = \$60$. We are then left with \$3, so we need 3 of the \$1 bills. Thus in total, we use:

$$21 \times \$100 \text{ bills}; \quad 6 \times \$10 \text{ bills}; \quad 3 \times \$1 \text{ bills}$$

This is a total of $21 + 6 + 3 = 30$ bills.

To give a player \$2163 using the greatest total number of bills, we want to use as many \$1 bills as possible. If we use all \$1 bills, then we will use 2163 bills in total.

(b) Using the strategy from (a) to use the fewest total number of bills, we can give \$254 as follows:

$$2 \times \$100 \text{ bills}; \quad 5 \times \$10 \text{ bills}; \quad 4 \times \$1 \text{ bills}$$

This uses a total of $2 + 5 + 4 = 11$ bills. Let's try replacing one \$100 bill with ten \$10 bills. This gives:

$$1 \times \$100 \text{ bill}; \quad 15 \times \$10 \text{ bills}; \quad 4 \times \$1 \text{ bills}$$

This uses a total of $1 + 15 + 4 = 20$ bills, so it is possible to give a player \$254 using exactly 20 bills in total.

Notice that every time we replace a bill with ten smaller bills, the total number of bills increases by 9. This is true if we replace one \$100 bill with ten \$10 bills or if we replace one \$10 bill with ten \$1 bills. So the total number of bills is a sequence that starts at 11 and increases by 9 each time, until it reaches 254 (which is the greatest total number of bills that can be used for \$254). Writing out more terms in the sequence gives 11, 20, 29, $38, \ldots$. Since 30 is not in this sequence, we can *not* give a player \$254 using exactly 30 bills in total.

SOLUTION TO EXTENSION: If a player gets 1 of each bill, then the total amount is $1 \times \$100 + 1 \times \$10 + 1 \times \$1 = \$111$. If a player gets 2 of each bill, then the total amount will be $2 \times \$111 = \$222$, because they have twice as many of each bill, so the total amount will double. Similarly, if a player gets 6 of each bill, then the total amount will be $6 \times \$111 = \$666$, and if a player gets 15 of each bill, then the total amount will be $15 \times \$111 = \$1665$. Thus, the total amount will always be a multiple of 111.

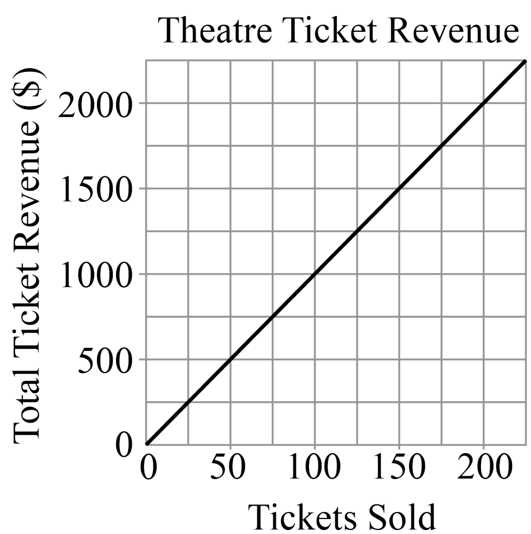
We can also use variables to explain this. Suppose a player receives n bills of each type. Then the total amount is equal to $(n \times 100 + n \times 10 + n \times 1)$. This is the same as $n \times (100 + 10 + 1)$, which equals $n \times 111$. Therefore, the total amount must be a multiple of 111.

Problem of the Week

Problem B

Let's Go to the Movies

At a movie theatre, all tickets are the same price. The *ticket revenue* is the money the theatre gets from customers when they buy tickets. The line graph shows the total ticket revenue when different amounts of tickets are sold.



(a) What is the total ticket revenue when 100 tickets are sold?

(b) How much does one ticket cost?

(c) The theatre has 250 seats in total. What is the total ticket revenue if they sell out?

(d) The theatre is planning an open air movie for which they will charge the same price per ticket. If the open air space can hold 600 people, what is the maximum total ticket revenue for that show?

(e) How can you tell from the graph that all tickets are the same price? Explain.

Themes: Algebra, Data Management

Problem of the Week
Problem B and Solution
Let's Go to the Movies

Problem

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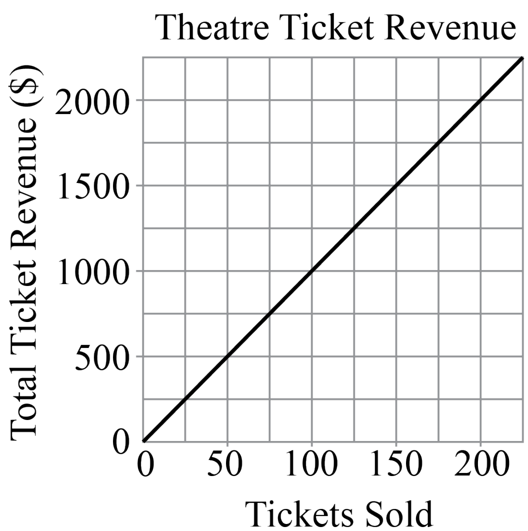

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(e) How can you tell from the graph that all tickets are the same price? Explain.

Solution

(a) If 100 tickets are sold, then we can use the graph to determine that the total ticket revenue will be \$1000.

(b) The cost for 1 ticket is the same as the total ticket revenue for 1 ticket. We can't easily read the revenue from the graph when 1 ticket is sold. However, we determined in (a) that the revenue for 100 tickets is \$1000. Since all tickets are the same price, then 1 ticket costs $\$1000 \div 100 = \$10$.

(c) Since we determined in (b) that one ticket costs \$10, then 250 tickets cost $250 \times \$10 = \$2500$. So the total ticket revenue will be \$2500 if they sell out.

Alternatively, we can extend the graph to 250 tickets sold by adding one more gridline to the right. When we extend the diagonal line, we will find that we also need to add one more gridline to the top, and that when we reach 250 tickets sold, the total ticket revenue will be \$2500.

(d) The total ticket revenue will be at its maximum when all 600 tickets are sold. Since we determined in (b) that one ticket costs \$10, then 600 tickets cost $600 \times \$10 = \$6000$. So the maximum total ticket revenue is \$6000.

Alternatively, we can use the graph to determine that when 200 tickets are sold, the total ticket revenue will be \$2000. Since $600 = 3 \times 200$, then the total ticket revenue when 600 tickets are sold is $3 \times \$2000 = \$6000$.

(e) The graph is a straight line, which means that as the number of tickets sold increases, the total ticket revenue increases at a constant rate. This means that each ticket must be the same price.

Computational Thinking (C)

Take me to the cover

Problem of the Week

Problem B

The Apartments are Multiplying

Alisa's apartment building has 12 floors, with 6 apartments on each floor. We have the following information about three other apartment buildings.

- Compared to Alisa's building, Bettina's apartment building has the same number of floors, but three times as many apartments on each floor.
- Compared to Bettina's building, Colin's apartment building has twice as many floors, but half as many apartments on each floor.
- Compared to Colin's building, Dara's apartment building has twice as many floors, and three times as many apartments on each floor.

Use this information to complete the following table. How does the total number of apartments change from one row to the next?

Building Owner	Number of Floors	Number of Apartments per Floor	Total Number of Apartments
Alisa	12	6	
Bettina			
Colin			
Dara			

EXTENSION: Ferid's apartment building has 20 apartments in total. Compared to Ferid's building, Gauri's apartment building has five times as many floors, but the same number of apartments per floor. How many apartments in total does Gauri's building have?

THEMES: COMPUTATIONAL THINKING, NUMBER SENSE

Problem of the Week

Problem B and Solution

The Apartments are Multiplying

Problem

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EXTENSION: Ferid's apartment building has 20 apartments in total. Compared to Ferid's building, Gauri's apartment building has five times as many floors, but the same number of apartments per floor. How many apartments in total does Gauri's building have?

Solution

The completed table is shown.

Building Owner	Number of Floors	Number of Apartments per Floor	Total Number of Apartments
Alisa	12	6	72
Bettina	12	18	216
Colin	24	9	216
Dara	48	27	1296

We notice the following about the total number of apartments.

- Bettina's apartment building has 3 times as many apartments as Alisa's. This is because the number of floors did not change but the number of apartments per floor was multiplied by 3, so the total number of apartments was multiplied by 3.

- Colin's apartment building and Bettina's apartment building have the same number of apartments. This is because the number of floors was multiplied by 2, but the number of apartments per floor was divided by 2. Since these are opposite operations, the total number of apartments did not change.
- Dara's apartment building has 6 times as many apartments as Colin's. This is because the number of floors was multiplied by 2, and the number of apartments per floor was multiplied by 3, so the total number of apartments was multiplied by $3 \times 2 = 6$.

Extension Solution:

Since Gauri's apartment building has 5 times as many floors, but the same number of apartments per floor as Ferid's, then the number of apartments in Gauri's building will be equal to 5 times the number of apartments in Ferid's building, which is $20 \times 5 = 100$.

Problem of the Week

Problem B

Book Club

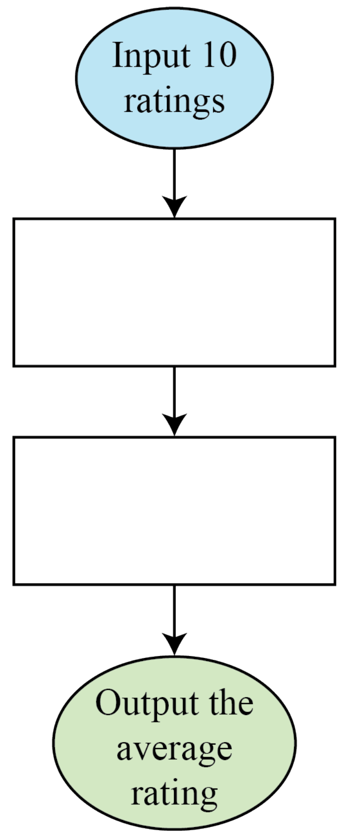
In Emil's book club, each of the 10 members rates each book they read on a scale from 1 to 5. Emil wants to write a program to calculate the average rating for a book.

(a) Complete the given flowchart to show the steps Emil's program needs to follow to calculate the average (mean) rating for a book.

(b) Emil finds that sometimes people make mistakes when typing their ratings. Modify your flowchart so that if a rating is not between 1 and 5, users have to input the 10 ratings again.

(c) Modify your flowchart from part (b) so that if the average rating is over 4, a message is displayed that says the book is excellent.

(d) What is the lowest possible rating that one of the book club members could give a book that could still result in a message saying the book is excellent?



Themes: Computational Thinking, Data Management

Problem of the Week
Problem B and Solution
Book Club

Problem

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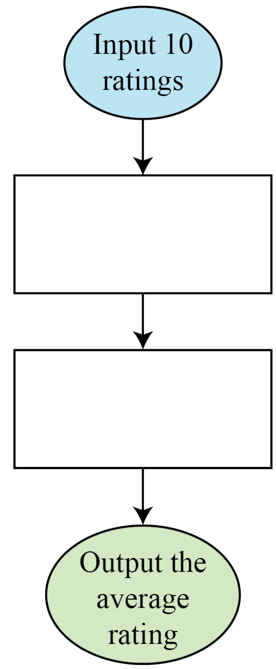
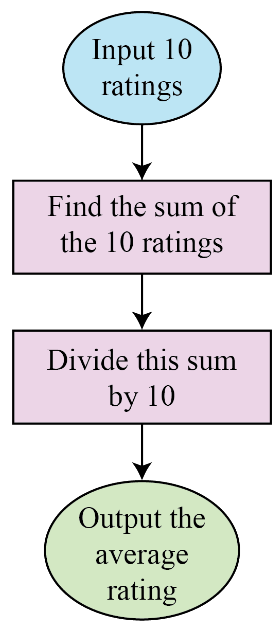
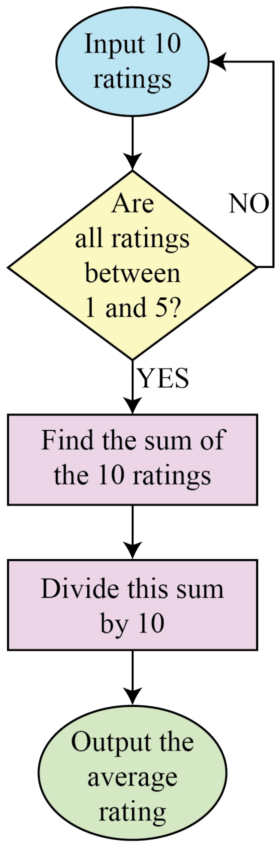


Solution

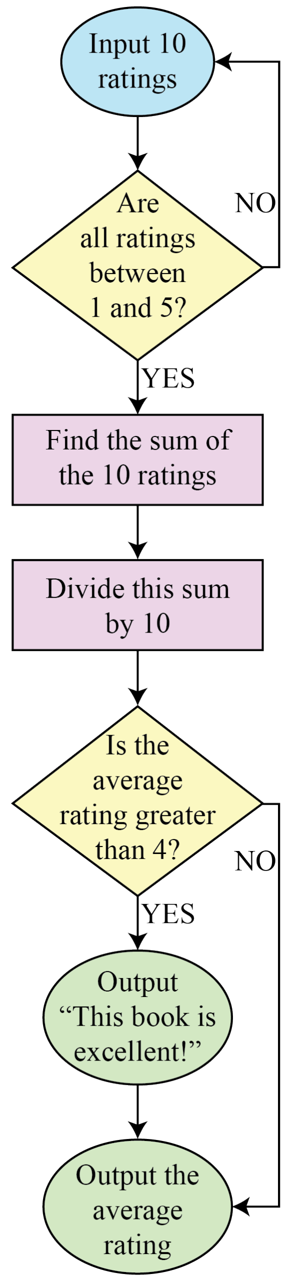
(a) In order to calculate the average rating, we first need to add the 10 ratings together, and then divide this value by 10. These steps are shown in the flowchart.



(b) The program should check for errors right after the 10 ratings are inputted. If the ratings are not all between 1 and 5, then the program should return to the beginning so the user can input the 10 ratings again. This is shown in the flowchart.



(c) The program needs check if the average rating is greater than 4 sometime after it has been calculated. In the flowchart shown, we first check if the average rating is greater than 4 and then output the average rating. However it is also possible to switch the order of these two steps.



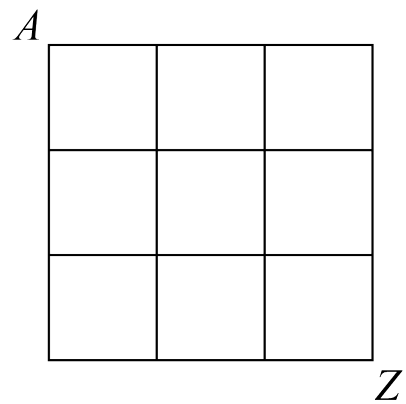
(d) If a message says the book is excellent, then it must have an average rating greater than 4. This means the sum of the 10 ratings must be greater than $4 \times 10 = 40$. Suppose 9 of the members gave a rating of 5. Then the sum of their ratings would be $9 \times 5 = 45$. So the remaining member could give a rating as low as 1 and the sum of all 10 ratings would still be greater than 40. Therefore, the lowest possible rating is 1.

Problem of the Week

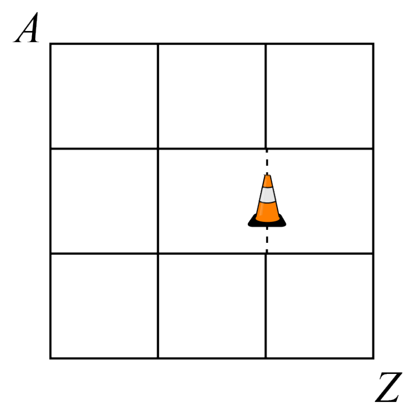
Problem B

Pathways

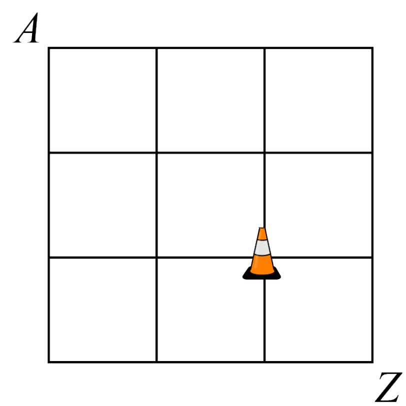
(a) Armaan walks to the zoo every day. A map of the streets between Armaan's house and the zoo is shown, where Armaan's house is represented by A, the zoo is represented by Z, and the streets are represented by line segments. How many different routes can Armaan take from his house to the zoo if he always walks either east or south? Consider the top of the page to be north.



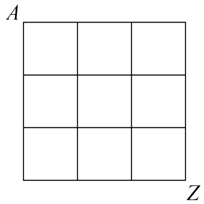
(b) On Tuesday there is some construction, so part of a street is closed, as shown. Armaan cannot walk on the closed part. How many different routes can Armaan take from his house to the zoo on Tuesday?



(c) On Friday, an intersection is closed, as shown. Armaan cannot walk through this intersection. How many different routes can Armaan take from his house to the zoo on Friday?



Themes: Computational Thinking, Geometry & Measurement



Problem of the Week

Problem B and Solution

Pathways

Problem

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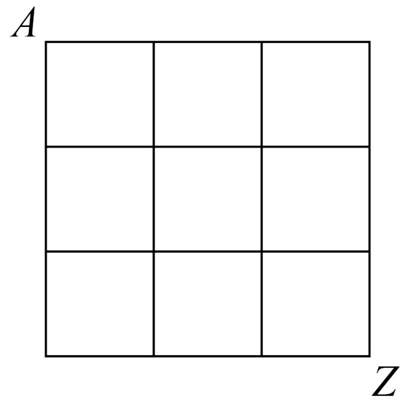

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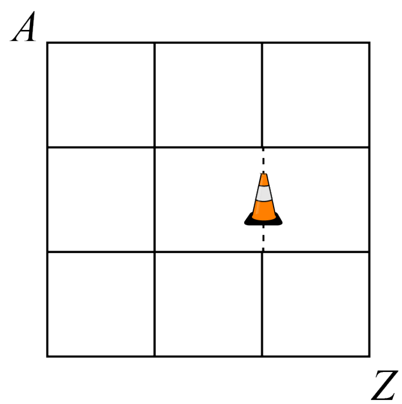

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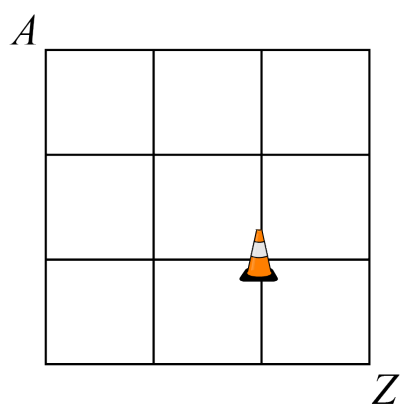
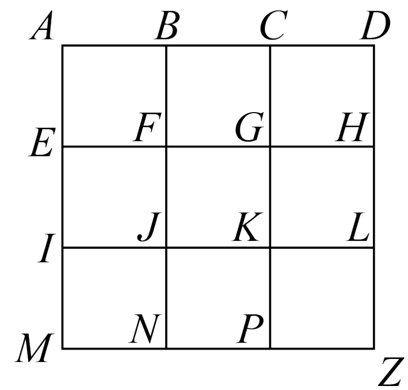
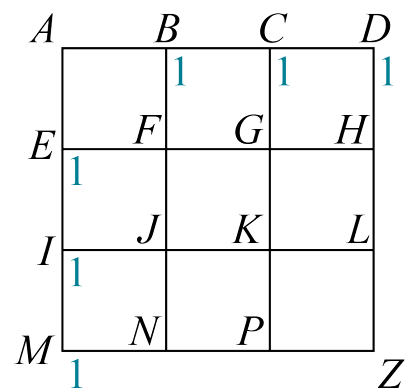


Solution

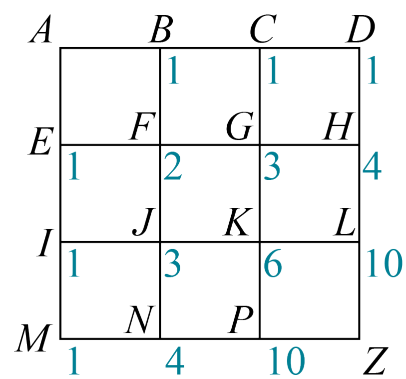
To count the number of routes, we could just draw as many routes as we can think of and see if we can find them all. This might work, but would take a long time, especially if the grid was larger. Here we show a more systematic way of solving the problem. First we will mark each intersection with a letter, as shown.



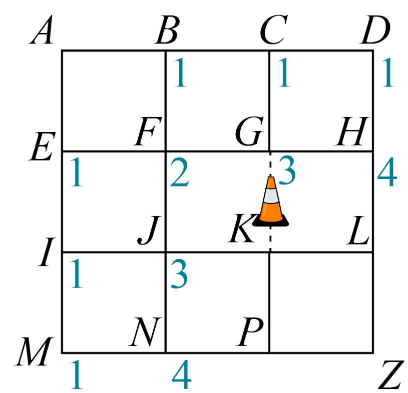
(a) From A, there is only 1 route to each of B, C, or D. Similarly there is only 1 route to each of E, I, and M. We update our diagram by indicating the number of routes from A to each of these intersections.



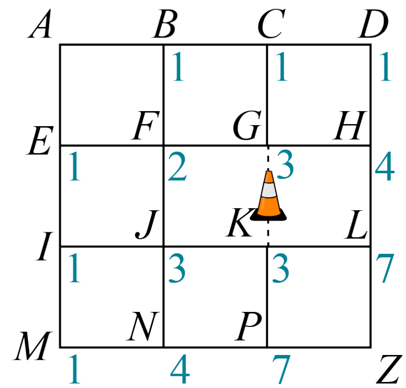
To reach F, Armaan would need to come from either B or E. Since there is 1 route to B and 1 route to E, it follows that there are $1 + 1 = 2$ routes to F. To reach G, Armaan would need to come from either F or C. Since there are 2 routes to F and 1 route to C, it follows that there are $2 + 1 = 3$ routes to G. Continuing in this way, by adding the number of routes to the preceding intersections, we can determine that there are 10 routes to P and 10 routes to L, so there are $10 + 10 = 20$ different routes from Armaan's house to the zoo.



(b) The counting of possible routes follows the same pattern as in (a), except for K and all intersections south and east of it, as shown.



To reach K, Armaan would need to come from J, so there are 3 routes to K. We can then continue counting as before, by adding the number of routes to the preceding intersections. Then we can determine that since there are 7 routes to P and 7 routes to L, there are $7 + 7 = 14$ different routes from Armaan's house to the zoo on Tuesday.



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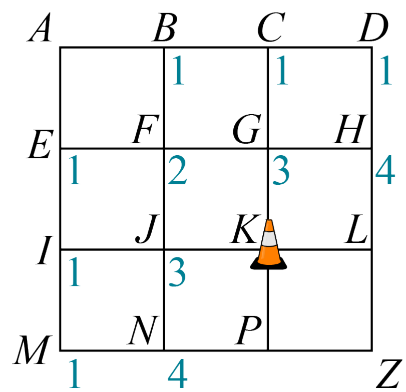
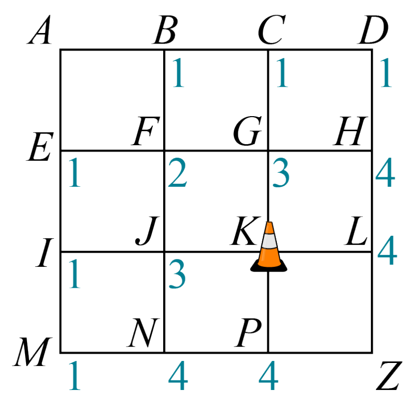


Armaan cannot reach K due to construction. To reach L, Armaan would need to come from H, so there are 4 routes to L. Similarly, to reach P, Armaan would need to come from N, so there are 4 routes to P. Since there are 4 routes to P and 4 routes to L, there are $4 + 4 = 8$ different routes from Armaan's house to the zoo on Friday.

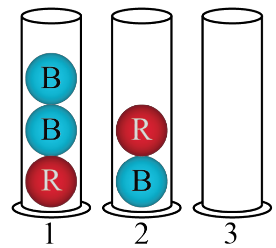


Problem of the Week

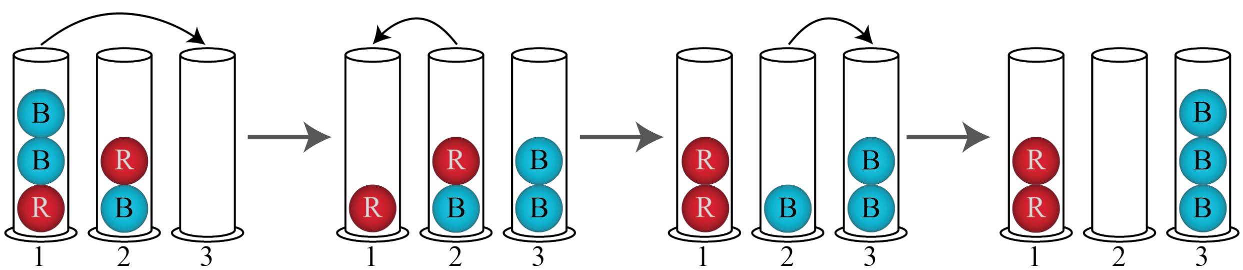
Problem B

Sorting Marbles

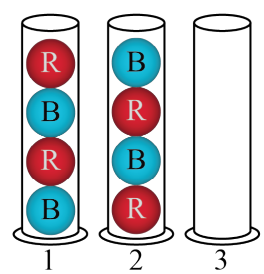
Dr. Cy Linder has tubes containing marbles of different colours which he would like to sort. He pours marbles from one tube to another until all the marbles of the same colour are in their own tube. For example, Dr. Linder starts with blue and red marbles in the three tubes shown.



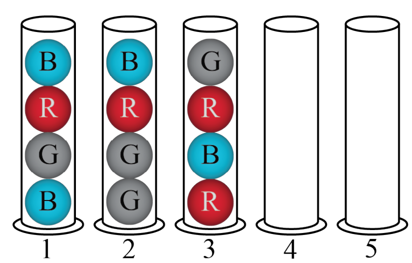
He can sort the marbles by first pouring both blue marbles from Tube 1 to Tube 3. Next he pours the red marble from Tube 2 to Tube 1. Finally he pours the blue marble from Tube 2 to Tube 3. The marbles are now sorted, and it took 3 pours in total. Note that this is not the only way to sort these marbles.



(a) Write down steps to sort the marbles in the given tubes. How many pours did you need in total?



(b) Write down steps to sort the marbles in the given tubes. How many pours did you need in total?



Extension: Create your own marble sorting problem that requires 5 pours to be sorted.

Theme: Computational Thinking

Problem of the Week
Problem B and Solution
Sorting Marbles

Problem

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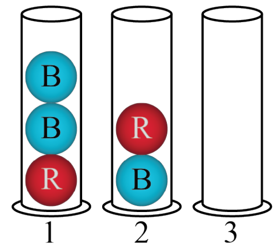

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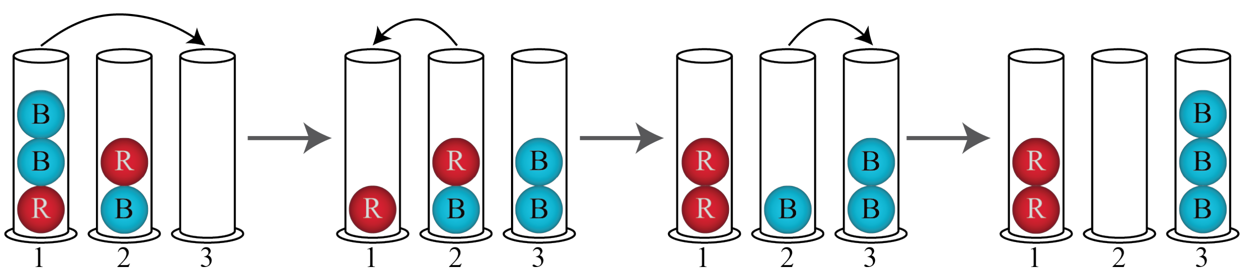
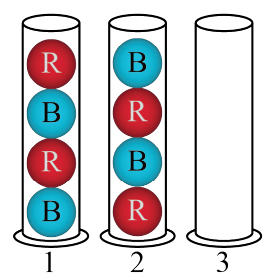
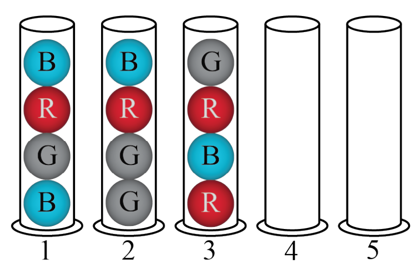


(a) Write down steps to sort the marbles in the given tubes. How many pours did you need in total?



(b) Write down steps to sort the marbles in the given tubes. How many pours did you need in total?



Extension: Create your own marble sorting problem that requires 5 pours to be sorted.

Solution

Students may find it helpful to to use coloured strips of paper or coloured cubes to model the marbles in the tubes. Note that other solutions are possible.

(a) Using the following 7 steps, we can sort the marbles.

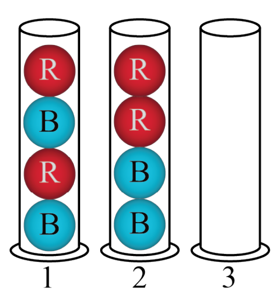
1. Pour the red marble from Tube 1 to Tube 3.
2. Pour the blue marble from Tube 2 to Tube 1.
3. Pour the red marble from Tube 2 to Tube 3.
4. Pour two blue marbles from Tube 1 to Tube 2.
5. Pour the red marble from Tube 1 to Tube 3.
6. Pour three blue marbles from Tube 2 to Tube 1. All the blue marbles are now in Tube 1.
7. Pour the red marble from Tube 2 to Tube 3. All the red marbles are now in Tube 3.

(b) Using the following 10 steps, we can sort the marbles.

1. Pour the blue marble from Tube 1 to Tube 4.
2. Pour the blue marble from Tube 2 to Tube 4.
3. Pour the red marble from Tube 1 to Tube 5.
4. Pour the red marble from Tube 2 to Tube 5.
5. Pour the grey marble from Tube 1 to Tube 2.
6. Pour the grey marble from Tube 3 to Tube 2. All the grey marbles are now in Tube 2.
7. Pour the blue marble from Tube 1 to Tube 4.
8. Pour the red marble from Tube 3 to Tube 5.
9. Pour the blue marble from Tube 3 to Tube 4. All the blue marbles are now in Tube 4.
10. Pour the red marble from Tube 3 to Tube 5. All the red marbles are now in Tube 5.

Solution to Extension:

Answers will vary. Here is one marble sorting problem that requires 5 pours to be solved. We leave it up to the reader to verify this.

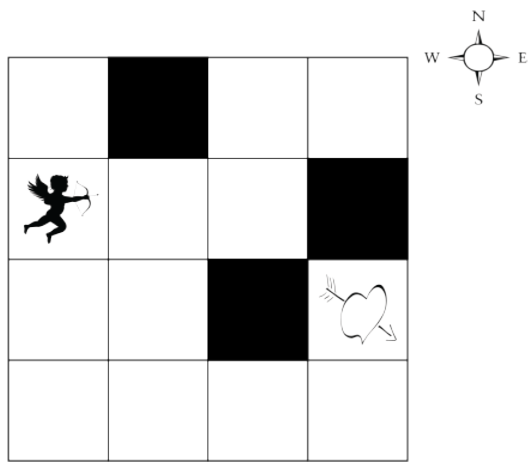


Problem of the Week

Problem B

Code to Guide Cupid

In the grid, the black squares represent obstacles to Cupid, who cannot go through them; nor can Cupid step outside the grid boundaries.



Let's play with some *pseudocode* to guide Cupid's path to the heart. The code will use the following instructions:

- fly1: moves Cupid one square in the current arrow direction
- rotc: turns (rotates) Cupid 90° clockwise
- rotcc: turns Cupid 90° counterclockwise

(a) For each set of pseudocode instructions, determine where Cupid ends up, or if an obstacle ends his quest (i.e., the code *crashes*).

(i)
```
fly1
rotc
fly1
fly1
rotc
fly1
```

(ii)
```
fly1
rotc
fly1
rotcc
fly1
```

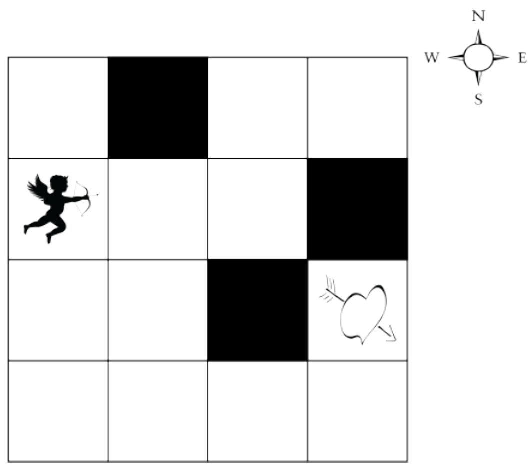
(b) Write pseudocode which guides Cupid to the heart.

Theme: Computational Thinking

Problem of the Week
Problem B and Solution
Code to Guide Cupid

Problem

In the grid, the black squares represent obstacles to Cupid, who cannot go through them; nor can Cupid step outside the grid boundaries.



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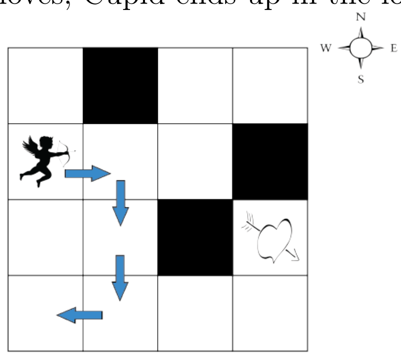
(i)
```
fly1
rotc
fly1
fly1
rotc
fly1
```

(ii)
```
fly1
rotc
fly1
rotcc
fly1
```

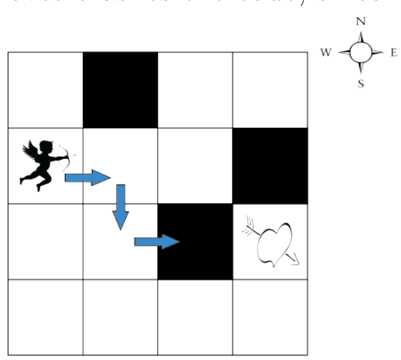
(b) Write pseudocode which guides Cupid to the heart.

Solution

(a) (i) We go through the pseudocode, moving Cupid as directed.
fly1: Cupid moves one square east
rotc: Cupid turns south
fly1: Cupid moves south one square
fly1: Cupid moves south one square
rotc: Cupid turns west
fly1: Cupid moves one square west
Following this sequence of moves, Cupid ends up in the lower left square on the grid.



(ii) We go through the pseudocode, moving Cupid as directed.
fly1: Cupid moves one square east
rotc: Cupid turns south
fly1: Cupid moves south one square
rotcc: Cupid turns east
fly1: Cupid moves east one square
Following this sequence of moves crashes the code, since Cupid is attempting to move through an obstacle.



(b) There are several possible sets of pseudocode. The two with the least number of instructions are given.

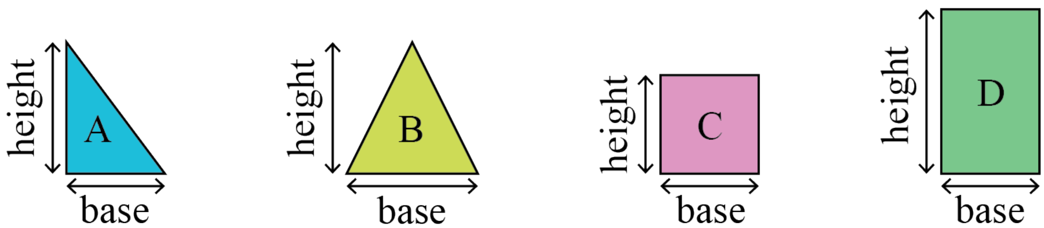
fly1	rotc
rotc	fly1
fly1	fly1
fly1	rotcc
rotcc	fly1
fly1	fly1
fly1	fly1
rotcc	rotcc
fly1	fly1

Problem of the Week

Problem B

Another Dimension

Kalle draws four shapes on grid paper. Shape A is a right-angled triangle, shape B is an isosceles triangle, shape C is a square, and shape D is a rectangle. Each shape has a horizontal base and a vertical height.



Using the following clues, determine the base and height for each shape.

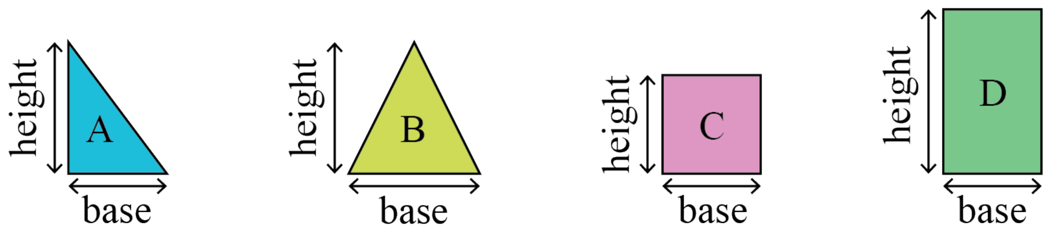
1. The base of shape A is equal to the base of shape D.
2. The base of shape A is one unit less than the base of shape B.
3. The height of shape C is equal to the base of shape A.
4. The height of shape B, the height of shape A, and the base of shape B are all equal.
5. The area of shape C is 9 square units.
6. The total area of all four shapes is 38 square units.

Themes: Computational Thinking, Geometry & Measurement

Problem of the Week
Problem B and Solution
Another Dimension

Problem

Kalle draws four shapes on grid paper. Shape A is a right-angled triangle, shape B is an isosceles triangle, shape C is a square, and shape D is a rectangle. Each shape has a horizontal base and a vertical height.



Using the following clues, determine the base and height for each shape.

1. The base of shape A is equal to the base of shape D.
2. The base of shape A is one unit less than the base of shape B.
3. The height of shape C is equal to the base of shape A.
4. The height of shape B, the height of shape A, and the base of shape B are all equal.
5. The area of shape C is 9 square units.
6. The total area of all four shapes is 38 square units.

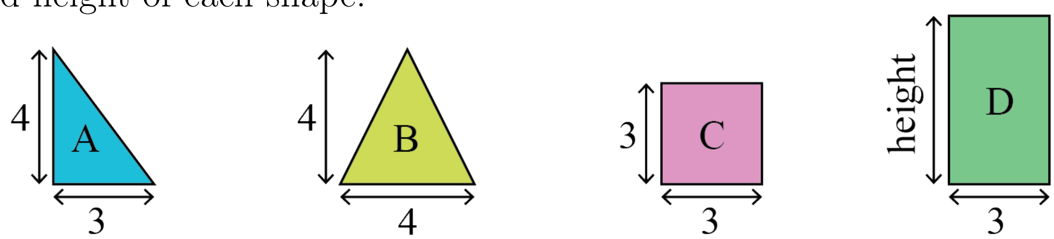
Solution

First we look at clue 5. Since the area of shape C is 9 square units and we know shape C is a square, then it must have a side length of 3 units, since $3 \times 3 = 9$. Thus, the base and height of shape C are each 3 units.

Then from clue 3 we can determine that the base of shape A is 3 units. Then from clue 1 we can determine that the base of shape D is also 3 units.

Then from clue 2 we can determine that the base of shape B must be one unit greater than the base of shape A. Thus, the base of shape B is $3 + 1 = 4$ units.

Then from clue 4 we can determine that the height of shape B and the height of shape A are also 4 units. We now fill in the information we know so far about the base and height of each shape.



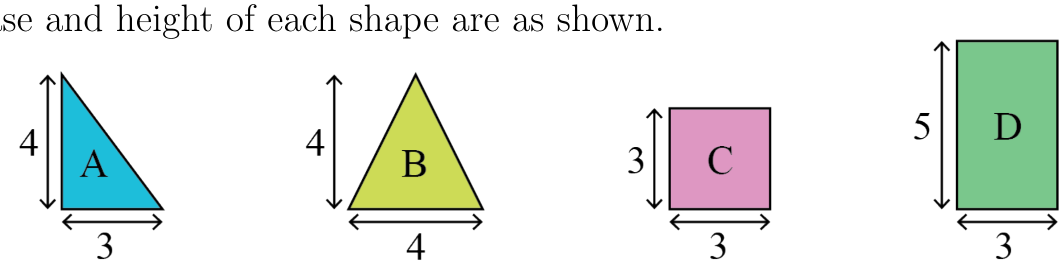
Thus, the only information we still need is the height of shape D. We can determine this using clue 6. First we will calculate the area of each shape.

- Shape A is a triangle, so its area is base $\times$ height $\div 2 = 3 \times 4 \div 2 = 12 \div 2 = 6$ square units.
- Shape B is a triangle, so its area is base $\times$ height $\div 2 = 4 \times 4 \div 2 = 16 \div 2 = 8$ square units.
- Shape C is a square, so its area is base $\times$ height $= 3 \times 3 = 9$ square units.

Thus, the total area of shapes A, B, and C is $6 + 8 + 9 = 23$ square units. Since the total area of all four shapes is 38 square meters, it follows that the area of shape D must be $38 - 23 = 15$ square units.

Shape D is a rectangle, so its area is base $\times$ height $= 3 \times$ height $= 15$ square units. It follows that its height must be 5 units since $3 \times 5 = 15$.

The base and height of each shape are as shown.



Data Management (D)

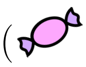
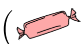
Take me to the cover

Problem of the Week

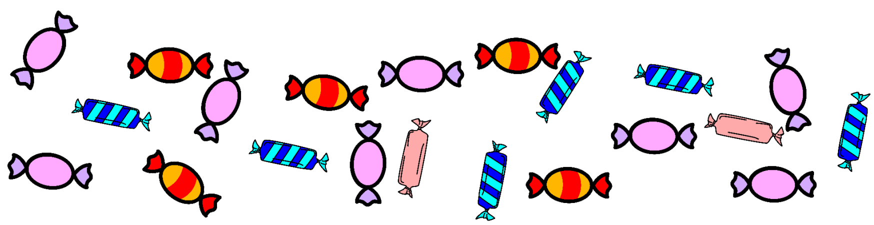
Problem B

A Jarring Thought

Chan and Mira are working at their school fun fair. They are responsible for filling a jar with candy, so people can guess how many candies are in the jar.

They buy 10 bags of candy. Each bag contains four types of candy, each with a different flavour: orange (), grape (), blueberry (), and watermelon ().

While they were filling the jar one of the bags broke open and spilled on the floor. The following is a picture of the spilled candy.



Chan and Mira enjoy a challenge, so they designed the following questions to be answered by their classmates.

(a) Each bag has the same distribution of candy as the bag that was spilled. Once all of the bags have been entered into the jar, what is the total number of each type of candy in the jar, and the total number of candies?

(b) After thoroughly mixing the candies, you stick your hand in the jar and randomly pull out one candy. What is the theoretical probability that it will be a blueberry candy?

(c) Chan found a bag of candy with the same number of candies as the other bags. However, this bag only had grape candies. They added these candies to the jar and the candies are thoroughly mixed. If you now randomly remove one candy from the jar, what is the theoretical probability that it will be a grape candy?

Theme: Data Management

Problem of the Week

Problem B and Solution

A Jarring Thought

Problem

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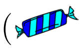
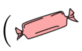
They buy 10 bags of candy. Each bag contains four types of candy, each with a different flavour: orange (), grape (), blueberry (), and watermelon ().

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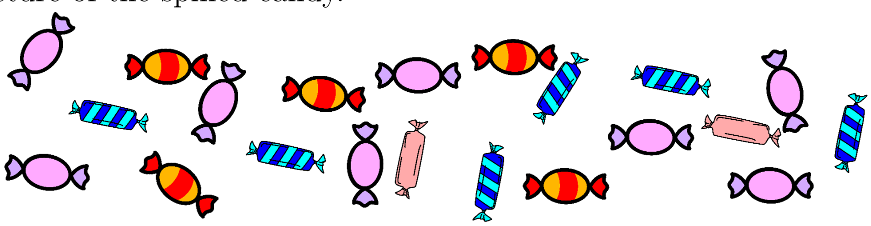

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Solution

(a) Since the entire quantity of candies in the jar is ten times what is shown, with the same distribution of types as shown, there will be ten times as many candies of each type. Since there are 5 orange candies, 8 grape candies, 6 blueberry candies, and 2 watermelon candies in one bag, then in total there are 50 orange candies, 80 grape candies, 60 blueberry candies, and 20 watermelon candies in the jar.
Thus, there are a total of $50 + 80 + 60 + 20 = 210$ candies in the jar.

(b) The theoretical probability that a candy will be a blueberry candy is $\frac{60}{210} = \frac{2}{7}$.
Alternatively, since the bags have the same ratio of the types of candy, we could have used the probability of pulling out a blueberry candy from the spilled bag, which is $\frac{6}{21} = \frac{2}{7}$.

(c) Since we are adding 21 grape candies to the jar, then there are a total of $80 + 21 = 101$ grape candies, and the total number of candies is $21 + 210 = 231$. Therefore, the theoretical probability that it will be a grape candy is $\frac{101}{231}$.

Problem of the Week

Problem B

Book Club

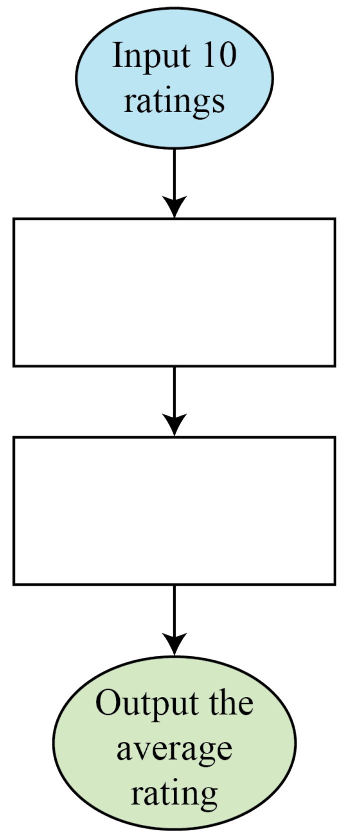
In Emil's book club, each of the 10 members rates each book they read on a scale from 1 to 5. Emil wants to write a program to calculate the average rating for a book.

(a) Complete the given flowchart to show the steps Emil's program needs to follow to calculate the average (mean) rating for a book.

(b) Emil finds that sometimes people make mistakes when typing their ratings. Modify your flowchart so that if a rating is not between 1 and 5, users have to input the 10 ratings again.

(c) Modify your flowchart from part (b) so that if the average rating is over 4, a message is displayed that says the book is excellent.

(d) What is the lowest possible rating that one of the book club members could give a book that could still result in a message saying the book is excellent?



Themes: Computational Thinking, Data Management

Problem of the Week
Problem B and Solution
Book Club

Problem

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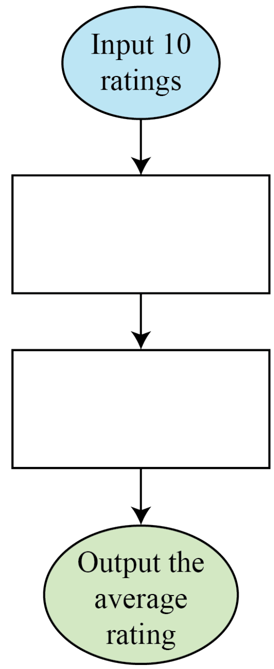
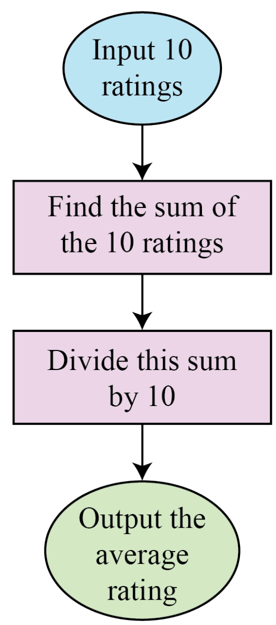
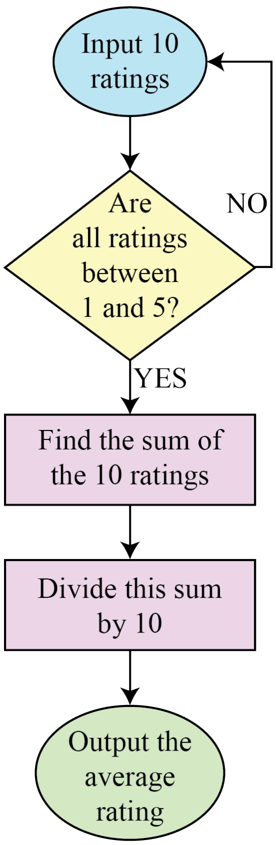


Solution

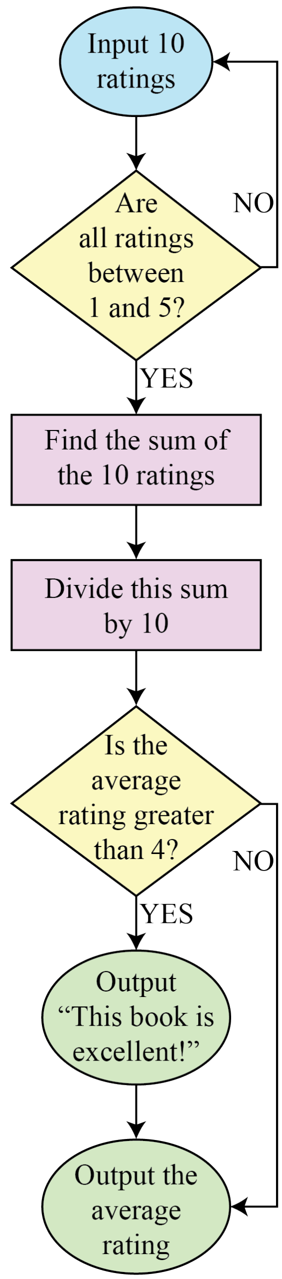
(a) In order to calculate the average rating, we first need to add the 10 ratings together, and then divide this value by 10. These steps are shown in the flowchart.



(b) The program should check for errors right after the 10 ratings are inputted. If the ratings are not all between 1 and 5, then the program should return to the beginning so the user can input the 10 ratings again. This is shown in the flowchart.



(c) The program needs check if the average rating is greater than 4 sometime after it has been calculated. In the flowchart shown, we first check if the average rating is greater than 4 and then output the average rating. However it is also possible to switch the order of these two steps.



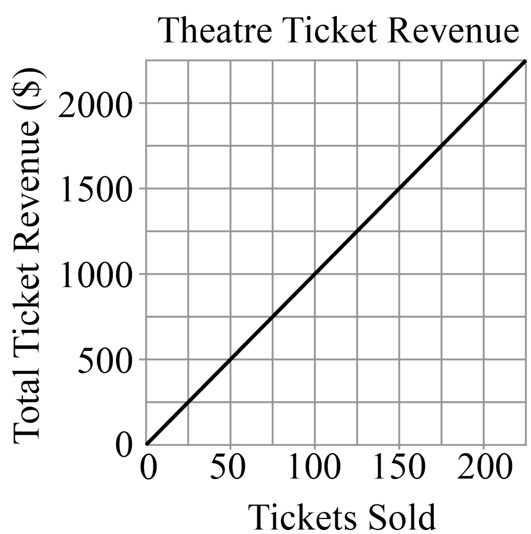
(d) If a message says the book is excellent, then it must have an average rating greater than 4. This means the sum of the 10 ratings must be greater than $4 \times 10 = 40$. Suppose 9 of the members gave a rating of 5. Then the sum of their ratings would be $9 \times 5 = 45$. So the remaining member could give a rating as low as 1 and the sum of all 10 ratings would still be greater than 40. Therefore, the lowest possible rating is 1.

Problem of the Week

Problem B

Let's Go to the Movies

At a movie theatre, all tickets are the same price. The *ticket revenue* is the money the theatre gets from customers when they buy tickets. The line graph shows the total ticket revenue when different amounts of tickets are sold.



(a) What is the total ticket revenue when 100 tickets are sold?

(b) How much does one ticket cost?

(c) The theatre has 250 seats in total. What is the total ticket revenue if they sell out?

(d) The theatre is planning an open air movie for which they will charge the same price per ticket. If the open air space can hold 600 people, what is the maximum total ticket revenue for that show?

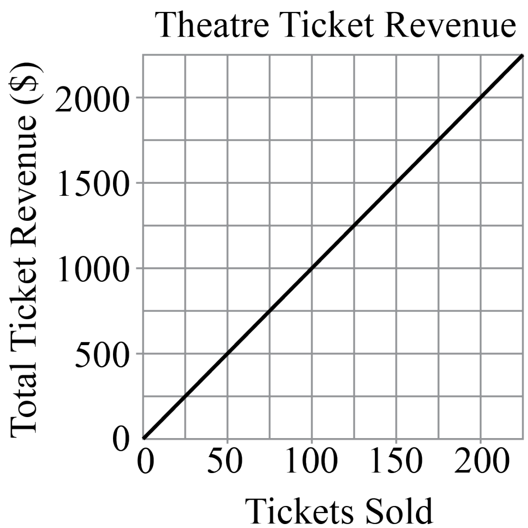
(e) How can you tell from the graph that all tickets are the same price? Explain.

Themes: Algebra, Data Management

Problem of the Week
Problem B and Solution
Let's Go to the Movies

Problem

At a movie theatre, all tickets are the same price. The *ticket revenue* is the money the theatre gets from customers when they buy tickets. The line graph shows the total ticket revenue when different amounts of tickets are sold.



(a) What is the total ticket revenue when 100 tickets are sold?

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(e) How can you tell from the graph that all tickets are the same price? Explain.

Solution

(a) If 100 tickets are sold, then we can use the graph to determine that the total ticket revenue will be \$1000.

(b) The cost for 1 ticket is the same as the total ticket revenue for 1 ticket. We can't easily read the revenue from the graph when 1 ticket is sold. However, we determined in (a) that the revenue for 100 tickets is \$1000. Since all tickets are the same price, then 1 ticket costs $\$1000 \div 100 = \$10$.

(c) Since we determined in (b) that one ticket costs \$10, then 250 tickets cost $250 \times \$10 = \$2500$. So the total ticket revenue will be \$2500 if they sell out.

Alternatively, we can extend the graph to 250 tickets sold by adding one more gridline to the right. When we extend the diagonal line, we will find that we also need to add one more gridline to the top, and that when we reach 250 tickets sold, the total ticket revenue will be \$2500.

(d) The total ticket revenue will be at its maximum when all 600 tickets are sold. Since we determined in (b) that one ticket costs \$10, then 600 tickets cost $600 \times \$10 = \$6000$. So the maximum total ticket revenue is \$6000.

Alternatively, we can use the graph to determine that when 200 tickets are sold, the total ticket revenue will be \$2000. Since $600 = 3 \times 200$, then the total ticket revenue when 600 tickets are sold is $3 \times \$2000 = \$6000$.

(e) The graph is a straight line, which means that as the number of tickets sold increases, the total ticket revenue increases at a constant rate. This means that each ticket must be the same price.

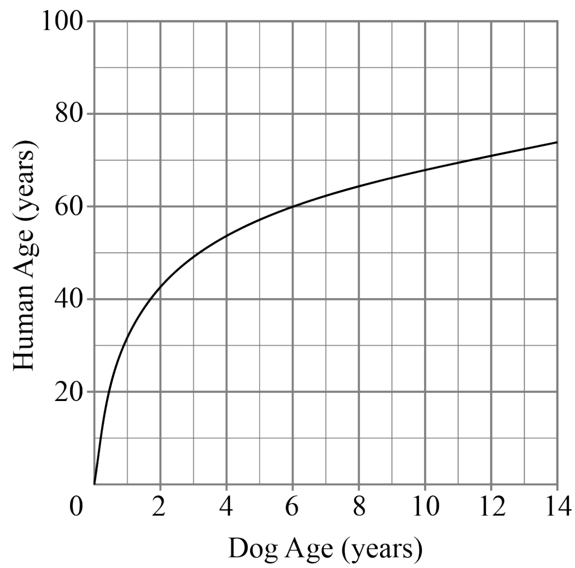
Problem of the Week

Problem B

It's Been A Dog's Age

A new way to relate a dog's age to a human's age, based on researchers studying labrador retrievers, is discussed at *www.caninejournal.com/dog-years-to-human-years*. The relationship they found is shown in the graph below.

Relationship Between Dog Age and Human Age



(a) A traditional way to relate a dog's age to a human's age is by multiplying the dog's age by 7. This comparison is called *linear* because its graph is a straight line. Since multiplication by 7 implies that 14 years of dog age equals $7 \times 14 = 98$ years of human age, this line will go from $(0, 0)$ to $(14, 98)$. Sketch this line carefully on the given graph.

(b) For dog ages of 2, 6, and 10 years, use your graph to estimate the human age predicted the traditional way and the new way.

(c) For what dog age are the two predicted human ages farthest apart? About how many years apart are the two predicted human ages?

(d) For what dog ages are the two predicted human ages the same?

(e) If the first year of a cat's life is equivalent to 15 human years, the second year to 9 human years, and each year thereafter to 4 human years, then show that by the age of 6 a cat will be younger in human years than either of the predicted dog ages in human years.

Theme: Data Management

Problem of the Week
Problem B and Solution
It's Been A Dog's Age

Problem

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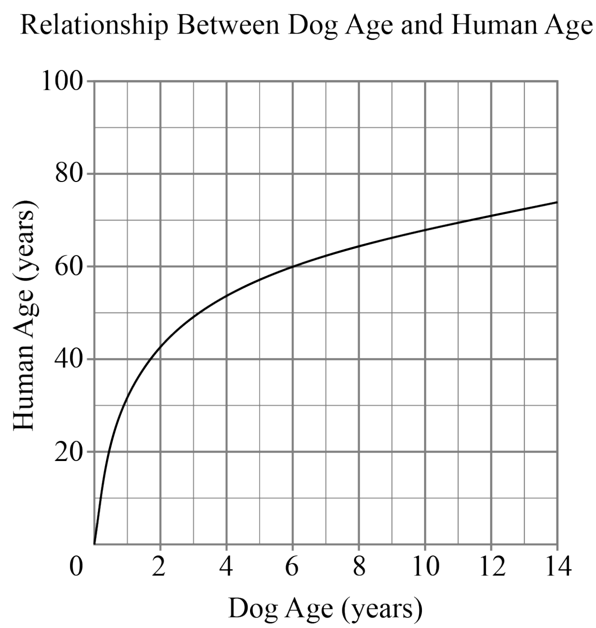

(a) A traditional way to relate a dog's age to a human's age is by multiplying the dog's age by 7. This comparison is called *linear* because its graph is a straight line. Since multiplication by 7 implies that 14 years of dog age equals $7 \times 14 = 98$ years of human age, this line will go from $(0, 0)$ to $(14, 98)$. Sketch this line carefully on the given graph.

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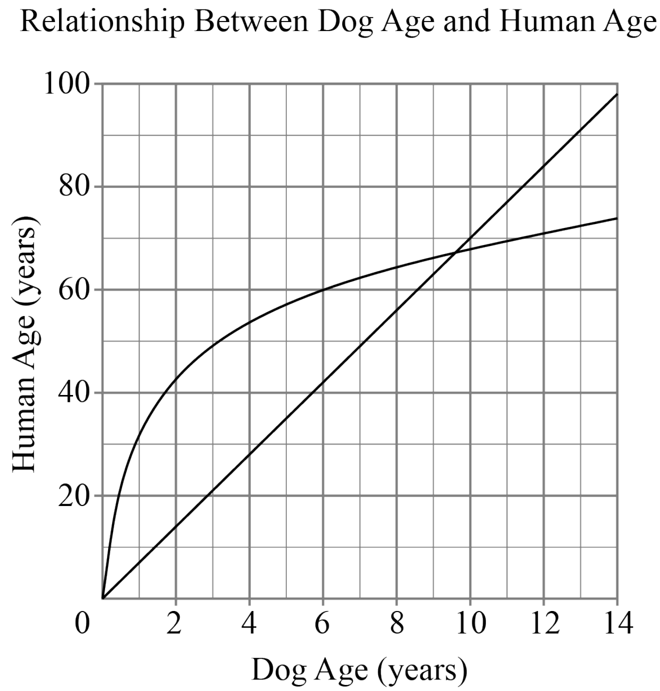
Solution

(a) The straight line representing the traditional 'multiply by 7' relationship is shown on the graph.



(b) At dog age 2, the linear graph predicts about 14 human years, while the curved graph predicts about 42 human years.
At dog age 6, the linear graph predicts about 42 human years, while the curved graph predicts about 60 human years.
At dog age 10, the linear graph predicts about 70 human years, while the curved graph predicts about 68 human years.

(c) By looking at the vertical distance between the two graphs, the difference between the two predicted human ages for a given dog age appears to be greatest at a dog age of about 2 years, at which the difference between the two predicted human ages is about $42 - 14 = 28$ years.

(d) The predicted human ages appear to be the same at about $9\frac{1}{2}$ years and at 0.

(e) Adding the first six years, the given data predicts that a cat of age 6 years will compare to a human at $15 + 9 + 4 + 4 + 4 + 4 = 40$ years. Since both dog predictions are greater than that (about 42 and 60), by the age of 6, the equivalent human years for a cat are less than those predicted for a dog.

To Think About: Will a cat remain younger in human years? How do you know?

Problem of the Week

Problem B

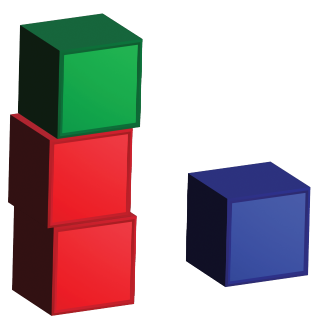
Sanjiv's Blocks

Priya's little brother Sanjiv loves to build towers with his coloured blocks.

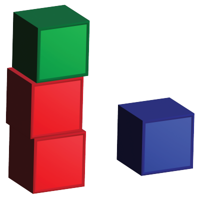
(a) Sanjiv builds a tower with three blocks: two identical red blocks and one green block. If each colour combination is equally likely, what is the theoretical probability that Sanjiv's tower will have the two red blocks next to each other?

(b) Sanjiv also builds towers with four blocks: two identical red blocks, one green block, and one blue block. If each colour combination is equally likely, what is the theoretical probability that such a tower will have the two red blocks next to each other?

(c) Priya notices that of the last six four-block towers Sanjiv built, four had the two red blocks next to each other. What is the experimental probability that one of Sanjiv's four-block towers has two red blocks next to each other? Does this observation confirm that Sanjiv does like to have the red blocks together?



Theme: Data Management



Problem of the Week
Problem B and Solution
Sanjiv's Blocks

Problem

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Solution

(a) Let R represent a red block and G represent a green block. There are three possible configurations of the blocks. Listing the blocks from top to bottom, the configurations are RRG, RGR, and GRR. Of these, two have the red blocks next to each other. Thus, the theoretical probability that Sanjiv's tower has two red blocks together is $\frac{2}{3}$.

(b) Let R represent a red block, G represent a green block, and B represent a blue block. There are twelve possible configurations of the blocks. Listing the blocks from top to bottom, the configurations are RRGB, RRBG, RGRB, RBRG, RGBR, RBGR, GRRB, BRRG, GRBR, BRGR, GBRR, BGRR. Of these, six have the red blocks next to each other. Thus, the theoretical probability that Sanjiv's tower has two red blocks together is $\frac{6}{12} = \frac{1}{2}$.

(c) Priya observes four of the six towers have the red blocks next to each other. Thus, the experimental probability that the two red blocks are next to each other is $\frac{4}{6} = \frac{2}{3}$. Since this is greater than the theoretical probability $\frac{1}{2}$, it is more likely to be Sanjiv's choice to have the red blocks next to each other. However, the sample size is quite small, and experimental and theoretical probabilities often differ, especially for small sample sizes.

Geometry & Measurement (G)

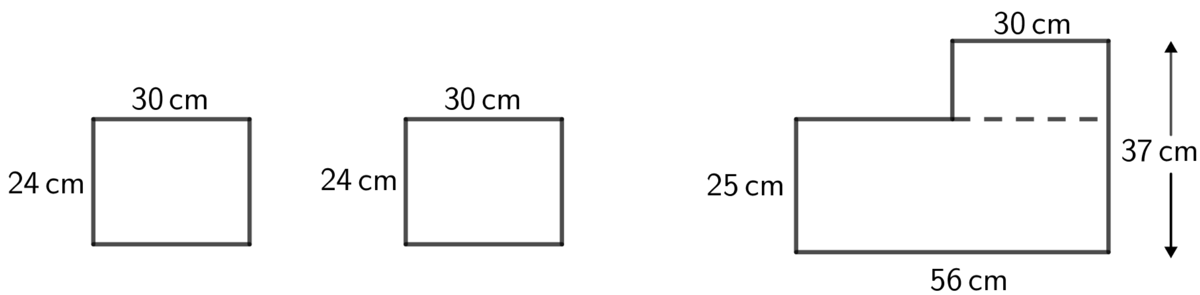
Take me to the cover

Problem of the Week

Problem B

Painting Stones

Iwa wishes to add three stepping stones to her garden. Two of the stepping stones are identical rectangular prisms and the third is a prism made up of two rectangular prisms. The dimensions of the top face of each of the three stepping stones are shown in the diagram below.



(a) Iwa wants to paint the top faces of the three stones. What is the area that she will paint?

(b) Iwa realizes that the side faces of the prisms will also be seen and, therefore, she wants to paint these as well. If these stones all have height of 6 cm, what is the total area that she will actually paint?

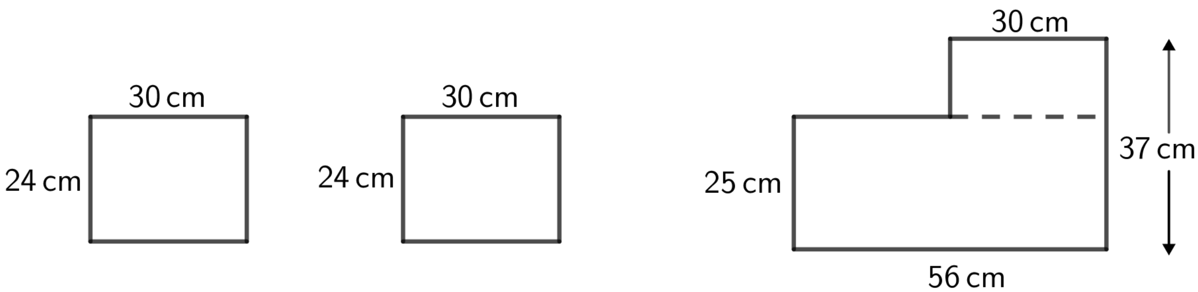
(c) She wants to put edging tightly around each of the stepping stones. What length of edging will she need?

Theme: Geometry & Measurement

Problem of the Week
Problem B and Solution
Painting Stones

Problem

Iwa wishes to add three stepping stones to her garden. Two of the stepping stones are identical rectangular prisms and the third is a prism made up of two rectangular prisms. The dimensions of the top face of each of the three stepping stones are shown in the diagram below.



(a) Iwa wants to paint the top faces of the three stones. What is the area that she will paint?

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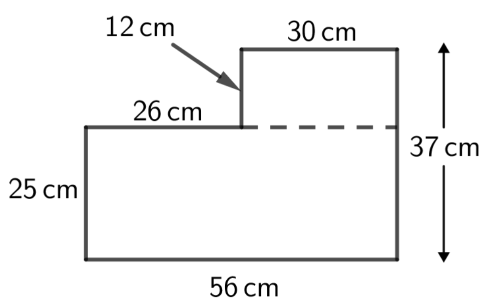
(c) She wants to put edging tightly around each of the stepping stones. What length of edging will she need?

Solution

(a) The two identical stones each have a top face with area $24 \times 30 = 720$ cm^2.

To determine the area of the top face of the third stone, we consider the two rectangles formed by the dashed line. The width of the smaller rectangle is equal to $37 - 25 = 12$ cm. The portion of the length of the larger rectangle which is not joined to the smaller rectangle is equal to $56 - 30 = 26$ cm.



Thus, the smaller rectangle has a length of 30 cm and a width of 12 cm, and so has area $30 \times 12 = 360$ cm^2.

The larger rectangle has a length of 56 cm and a width of 25 cm, and so has area $56 \times 25 = 1400$ cm^2.

The area of the top face of the third stepping stone is the sum of these two areas or $360 + 1400 = 1760$ cm^2.

Therefore, the total area painted is $720 + 720 + 1760 = 3200$ cm^2.

(b) The two stones that are rectangular prisms each have four side faces with dimensions 24 cm by 6 cm, 30 cm by 6 cm, 24 cm by 6 cm, and 30 cm by 6 cm.

Thus, the areas of the side faces are $24 \times 6 = 144$ cm^2, $30 \times 6 = 180$ cm^2, $24 \times 6 = 144$ cm^2, and $30 \times 6 = 180$ cm^2, respectively.

Therefore, the total area of the four side faces of one of these stones is $144 + 180 + 144 + 180 = 648$ cm^2.

The third stone composed of two rectangular prisms has six side faces with dimensions 30 cm by 6 cm, 37 cm by 6 cm, 56 cm by 6 cm, 25 cm by 6 cm, 26 cm by 6 cm, and 12 cm by 6 cm.

Thus, the areas of the side faces are $30 \times 6 = 180$ cm^2, $37 \times 6 = 222$ cm^2, $56 \times 6 = 336$ cm^2, $25 \times 6 = 150$ cm^2, $26 \times 6 = 156$ cm^2, and $12 \times 6 = 72$ cm^2, respectively.

Therefore, the total area of the six side faces of this stone is $180 + 222 + 336 + 150 + 156 + 72 = 1116$ cm^2.

Therefore, the total area that she will now paint is $3200 + 648 + 648 + 1116 = 5612$ cm^2.

(c) The length of edging needed is equal to the total perimeter of the top (or bottom) faces of the stepping stones.

The perimeter of the top face of one of the identical stones is equal to $24 + 30 + 24 + 30 = 108$ cm.

The perimeter of the top face of the third stone is equal to $30 + 37 + 56 + 25 + 26 + 12 = 186$ cm.

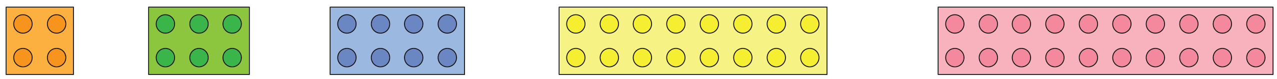
Therefore, the amount of edging needed is equal to $108 + 108 + 186 = 402$ cm.

Problem of the Week

Problem B

Playing with Bricks

Saskia has the following five sizes of Lego™ bricks.

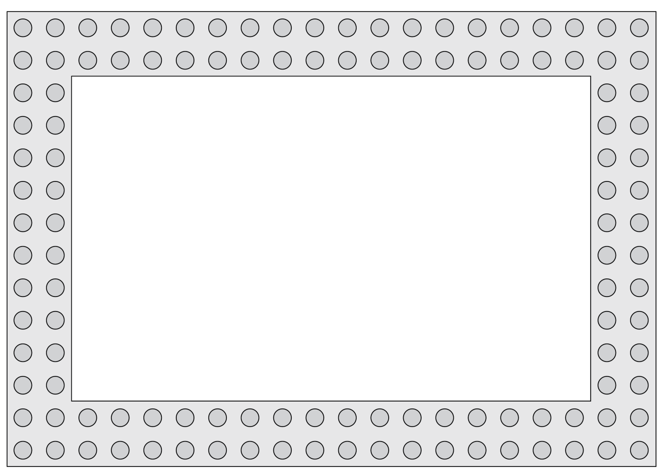


2×2 2×3 2×4 2×8 2×10

For each question below, assume that Saskia will never run out of bricks.

(a) Saskia wants to make a row of bricks that measures 2×16. Which of her brick sizes can she use if all bricks used must be the same size?

(b) Saskia wants to make a rectangular frame of bricks that measures 14×20 on the outside, as shown. Which of her brick sizes can she use if all bricks used must be the same size?



(c) Saskia wants to make a rectangular frame of bricks that measures 320×420 on the outside. What is the largest brick size that she can use if all bricks used must be the same size?

Themes: Geometry & Measurement, Number Sense

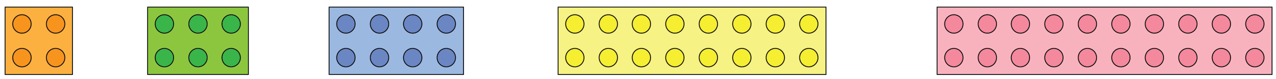
Problem of the Week

Problem B and Solution

Playing with Bricks

Problem

Saskia has the following five sizes of Lego™ bricks.



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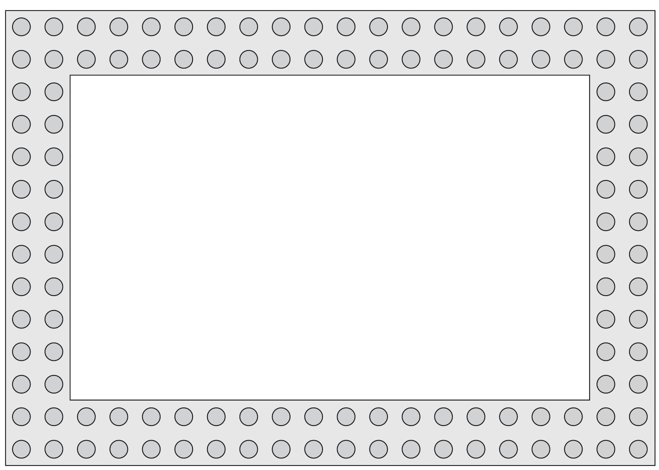


(c) Saskia wants to make a rectangular frame of bricks that measures 320×420 on the outside. What is the largest brick size that she can use if all bricks used must be the same size?

Solution

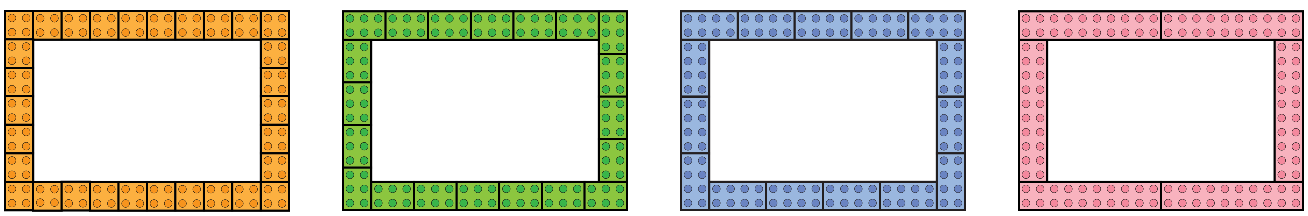
In our solutions, we call the smaller number in the brick size the *width*, and call the larger number the *length*. So a 2×4 brick has a width of 2 and a length of 4.

(a) Saskia can make a row that measures 2×16 by using eight 2×2 bricks, four 2×4 bricks, or two 2×8 bricks.

In general, since the width of the row is the same as the width of each brick, we can focus only on the lengths. If we want to place some number of the same brick in a row and have the total length equal 16, then the possible bricks are ones whose length is a factor of 16. These are 2×2, 2×4, and 2×8.

(b) We can think of the rectangular frame as two 2×20 rows on the top and bottom, and two 2×10 rows on the sides. If we placed these rows end to end, they would create a row with length $20 + 20 + 10 + 10 = 60$. Since 8 is the only length that is not a factor of 60, we know for sure that it is not possible to make the frame using only 2×8 bricks.

For the other brick sizes, we need to show that it is possible to place them around the frame. For each brick size, we start at the top-left corner and place bricks horizontally moving to the right. Once we can no longer place bricks horizontally, we start placing the bricks vertically, moving down. We continue in this clockwise fashion, until all bricks are placed around the frame. This method works for the 2×2, 2×3, 2×4, and 2×10 bricks, as shown.



2×2 $\quad$ 2×3 $\quad$ 2×4 $\quad$ 2×10

Thus, the possible brick sizes are 2×2, 2×3, 2×4, and 2×10.

(c) We can think of the rectangular frame as two 2×320 rows on the sides, and two 2×416 rows on the top and bottom. If we placed these rows end to end, they would create a row with length $320 + 320 + 416 + 416 = 1472$.

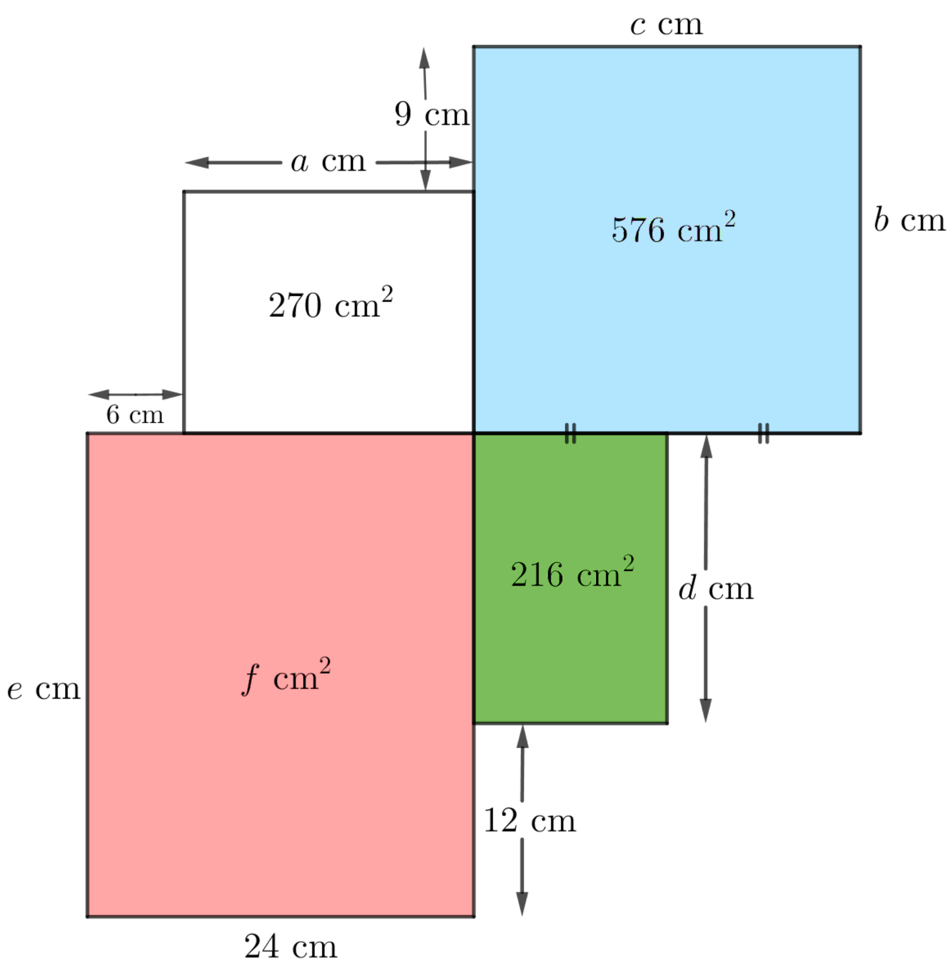
Since 10 is not a factor of 1472, we know it is not possible to make the frame using only 2×10 bricks. However, 8 is a factor of 1472, so we will attempt to place the 2×8 bricks around the frame. It helps to notice that 8 is a factor of both 320 and 416. Thus, we can make two 2×320 rows and two 2×416 rows out of 2×8 bricks. Putting these four rows together, with the 2×320 rows on the sides, and the 2×416 rows on the top and bottom gives the desired frame. Thus, the largest brick size that Saskia can use is 2×8.

Problem of the Week

Problem B

Puzzling Areas

Four rectangles meet at a common vertex, as shown in the diagram.



Determine the values of a, b, c, d, e, and f.

Theme: Geometry & Measurement

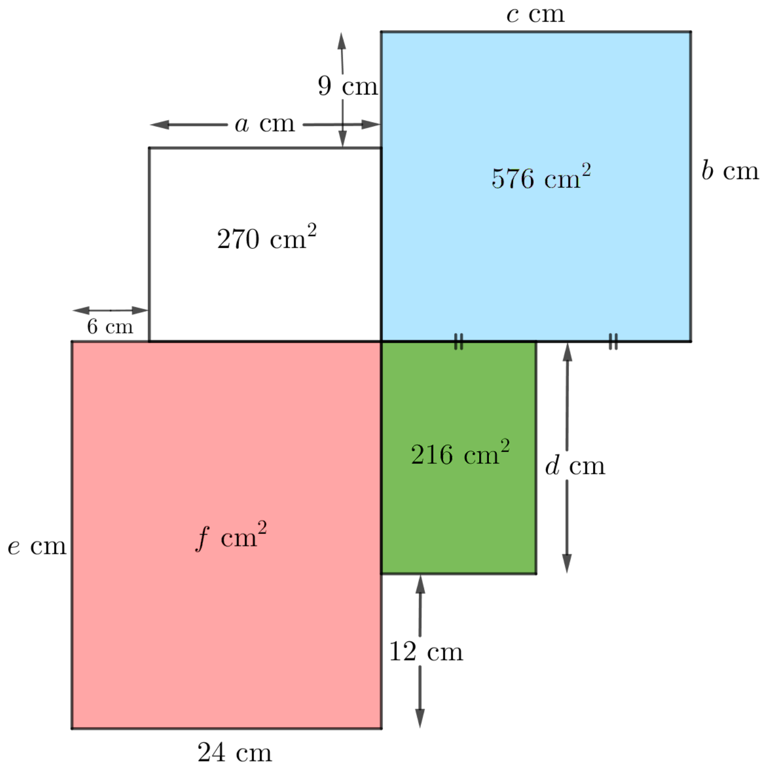
Problem of the Week

Problem B and Solution

Puzzling Areas

Problem

Four rectangles meet at a common vertex, as shown in the diagram.



Determine the values of a, b, c, d, e, and f.

Solution

We will call the top left rectangle A, the top right rectangle B, the bottom right rectangle C, and the bottom left rectangle D.

Since the length of the bottom side of D is 24 cm, the length of the top side of D is also 24 cm. Hence, the length of the bottom side of A is $24 - 6 = 18$ cm. Therefore, the length of the top side of A is 18 cm and so $a = 18$.

Since the area of A is 270 cm^2 and the length of the bottom side of A is 18 cm, then the length of the right side of A is $270 \div 18 = 15$ cm. Therefore, $b = 9 + 15 = 24$.

Since the area of B is 576 cm^2 and $b = 24$, then $c = 576 \div 24 = 24$.

Since the length of the top side of B is 24 cm, then the length of the bottom side is also 24 cm. Thus, the length of the top side of C is half of 24 cm or 12 cm. Since the area of C is 216 cm^2, then $d = 216 \div 12 = 18$.

Thus, $e = d + 12 = 18 + 12 = 30$.

Finally, the area of D is equal to $24 \times e = 24(30) = 720$ cm^2 and so $f = 720$.

Note: this puzzle is adapted from *"Area Mazes"* by Naoki Inaba.

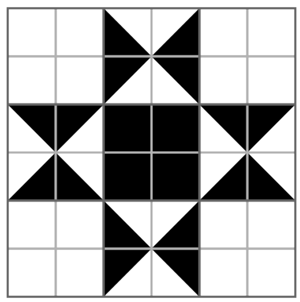
Problem of the Week

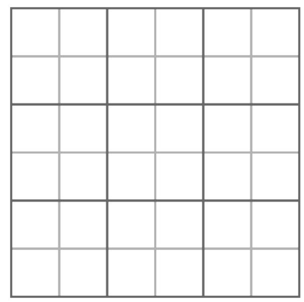
Problem B

Quilt Designs for Barns

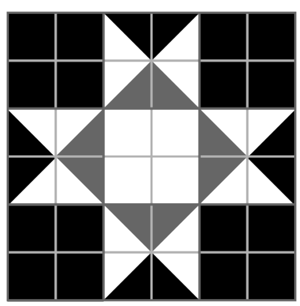
Sometimes people decorate their barns with quilts. These barn quilts are often designed in a square grid with a variety of colours and patterns, and they usually have symmetry in such a way that if they're rotated they will look the same. In the 6 by 6 grids below, you are going to explore and create some barn quilt designs. In each part, you can add straight lines as needed.

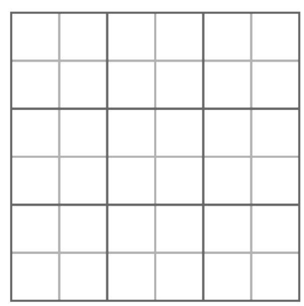
(a) What fraction of the barn quilt design below is black, and what fraction is white? In the blank grid, design another barn quilt using two colours and the same fractions.



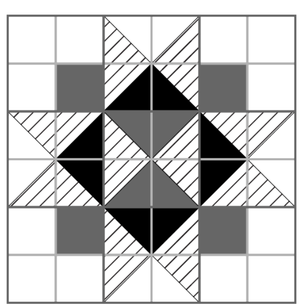


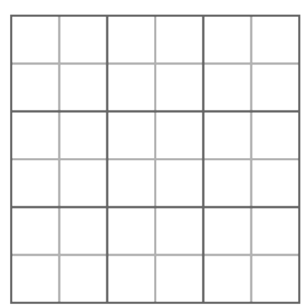
(b) What fraction of the barn quilt design below is black? Grey? White? In the blank grid, design another barn quilt using three colours and the same fractions.





(c) What fraction of the following barn quilt design below is black? Grey? Striped? White? In the blank grid, design another barn quilt using four colours and the same fractions.





Themes: Geometry & Measurement, Number Sense

Problem of the Week

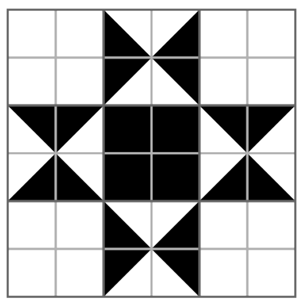
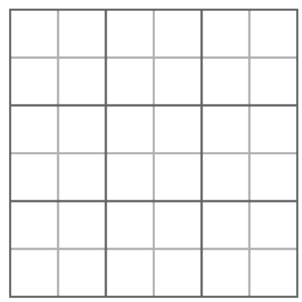
Problem B and Solution

Quilt Designs for Barns

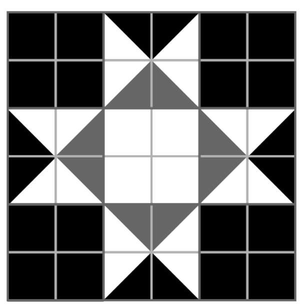
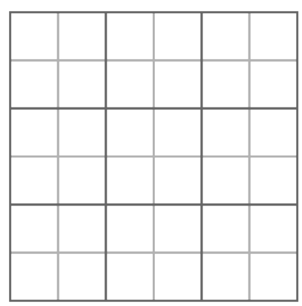
Problem

Sometimes people decorate their barns with quilts. These barn quilts are often designed in a square grid with a variety of colours and patterns, and they usually have symmetry in such a way that if they're rotated they will look the same. In the 6 by 6 grids below, you are going to explore and create some barn quilt designs. In each part, you can add straight lines as needed.

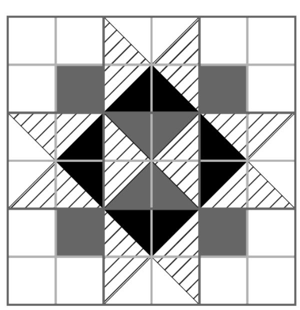
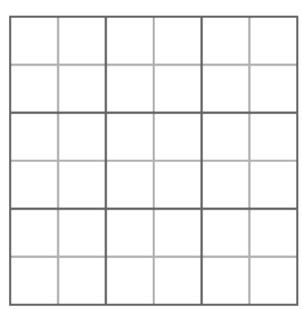
(a) What fraction of the barn quilt design below is black, and what fraction is white? In the blank grid, design another barn quilt using two colours and the same fractions.

(b) What fraction of the barn quilt design below is black? Grey? White? In the blank grid, design another barn quilt using three colours and the same fractions.

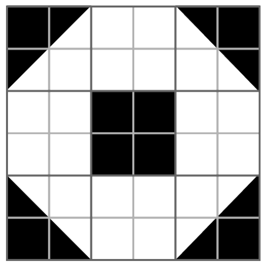
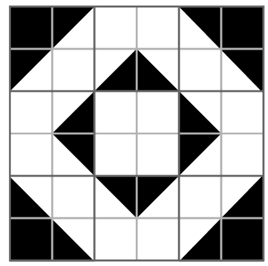
(c) What fraction of the following barn quilt design below is black? Grey? Striped? White? In the blank grid, design another barn quilt using four colours and the same fractions.

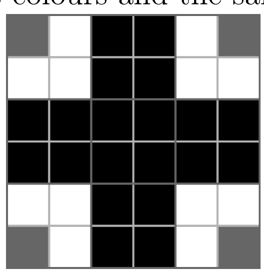
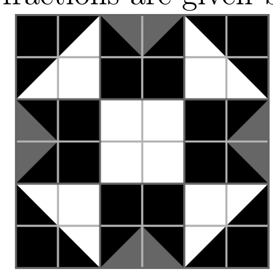
Solution

Note that a 6 by 6 grid consists of 36 smaller squares. There are also smaller triangles in the designs which each make up half of a smaller square. (These smaller triangles are formed when the two diagonals of a 2 by 2 square in the grid are drawn.)

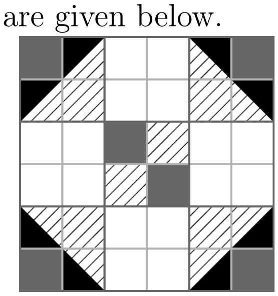
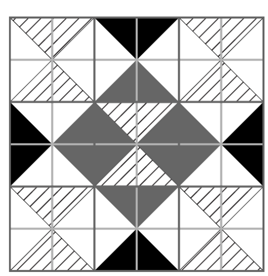
(a) The given barn quilt design has 16 white smaller squares plus 16 white smaller triangles, equivalent to $16 \div 2 = 8$ smaller squares. Therefore, the amount that is white is equivalent to $16 + 8 = 24$ smaller squares, and the fraction that is white is $\frac{24}{36} = \frac{2}{3}$. Thus, the fraction that is black must be $1 - \frac{2}{3} = \frac{1}{3}$. Two barn quilt designs using two colours and the same fractions are given below.

(b) The given barn quilt design has 16 black smaller squares plus 8 black smaller triangles, equivalent to $8 \div 2 = 4$ smaller squares. Therefore, the amount that is black is equivalent to $16 + 4 = 20$ smaller squares, and the fraction that is black is $\frac{20}{36} = \frac{5}{9}$. There are 8 grey smaller triangles, equivalent to a total of $8 \div 2 = 4$ grey smaller squares. Therefore, the fraction that is grey is $\frac{4}{36} = \frac{1}{9}$ and the fraction of white is $1 - \frac{5}{9} - \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$. Two barn quilt designs using three colours and the same fractions are given below.

(c) The given barn quilt design has 12 white smaller squares plus 8 white smaller triangles, equivalent to $8 \div 2 = 4$ smaller squares. Therefore, the amount that is white is equivalent to $12 + 4 = 16$ white smaller squares, and the fraction that is white is $\frac{16}{36} = \frac{4}{9}$. There are 8 black smaller triangles, equivalent to a total of $8 \div 2 = 4$ black smaller squares. Therefore, the fraction that is black is $\frac{4}{36} = \frac{1}{9}$. There are 4 smaller grey squares plus 4 grey smaller triangles, equivalent to $4 \div 2 = 2$ smaller squares. Therefore, the amount that is grey is equivalent to $4 + 2 = 6$ grey smaller squares, and the fraction that is grey is $\frac{6}{36} = \frac{1}{6}$. There are 20 striped smaller triangles, equivalent to $20 \div 2 = 10$ striped smaller squares. Therefore, the fraction that is striped is $\frac{10}{36} = \frac{5}{18}$. Two barn quilt designs using four colours and the same fractions are given below.

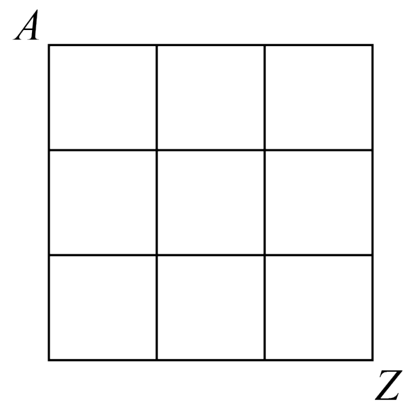



Problem of the Week

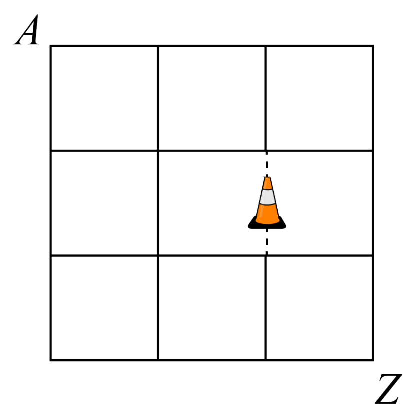
Problem B

Pathways

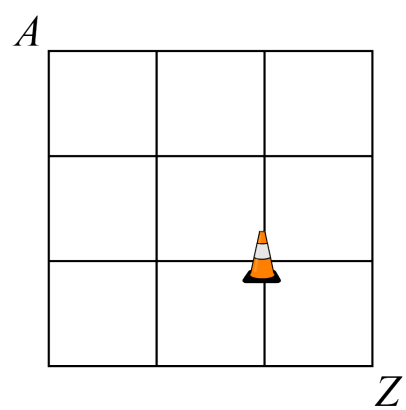
(a) Armaan walks to the zoo every day. A map of the streets between Armaan's house and the zoo is shown, where Armaan's house is represented by A, the zoo is represented by Z, and the streets are represented by line segments. How many different routes can Armaan take from his house to the zoo if he always walks either east or south? Consider the top of the page to be north.



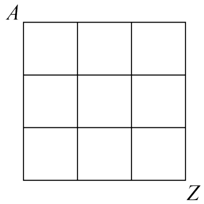
(b) On Tuesday there is some construction, so part of a street is closed, as shown. Armaan cannot walk on the closed part. How many different routes can Armaan take from his house to the zoo on Tuesday?



(c) On Friday, an intersection is closed, as shown. Armaan cannot walk through this intersection. How many different routes can Armaan take from his house to the zoo on Friday?



Themes: Computational Thinking, Geometry & Measurement



Problem of the Week

Problem B and Solution

Pathways

Problem

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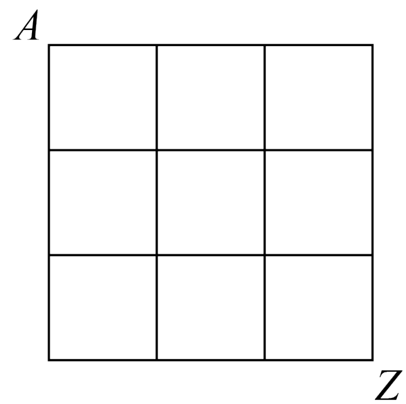

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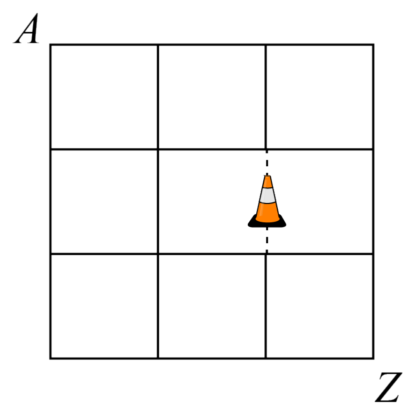
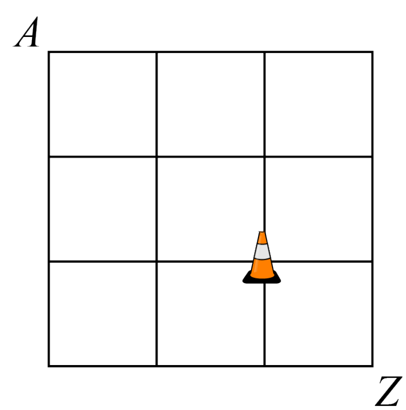
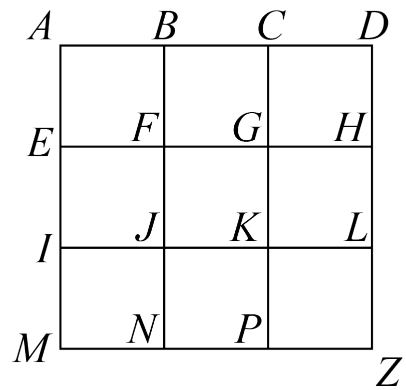


(c) On Friday, an intersection is closed, as shown. Armaan cannot walk through this intersection. How many different routes can Armaan take from his house to the zoo on Friday?

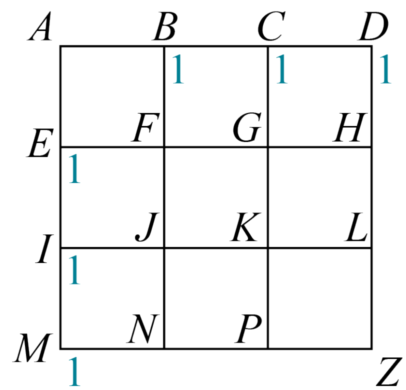


Solution

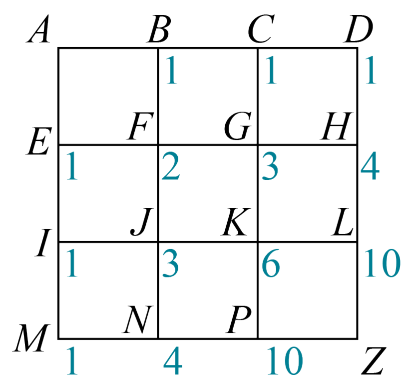
To count the number of routes, we could just draw as many routes as we can think of and see if we can find them all. This might work, but would take a long time, especially if the grid was larger. Here we show a more systematic way of solving the problem. First we will mark each intersection with a letter, as shown.



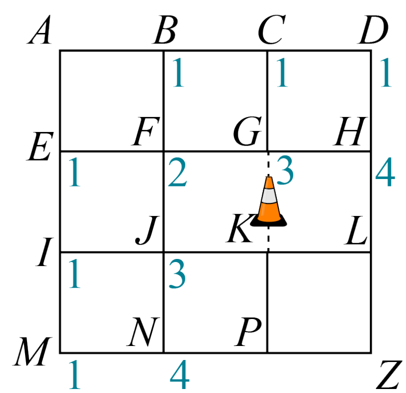
(a) From A, there is only 1 route to each of B, C, or D. Similarly there is only 1 route to each of E, I, and M. We update our diagram by indicating the number of routes from A to each of these intersections.



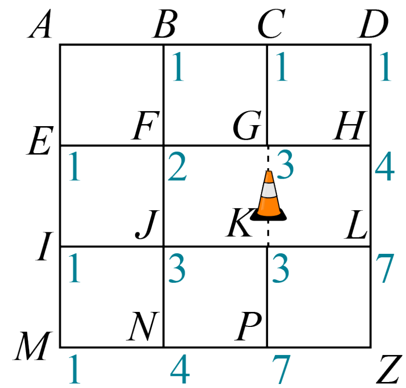
To reach F, Armaan would need to come from either B or E. Since there is 1 route to B and 1 route to E, it follows that there are $1 + 1 = 2$ routes to F. To reach G, Armaan would need to come from either F or C. Since there are 2 routes to F and 1 route to C, it follows that there are $2 + 1 = 3$ routes to G. Continuing in this way, by adding the number of routes to the preceding intersections, we can determine that there are 10 routes to P and 10 routes to L, so there are $10 + 10 = 20$ different routes from Armaan's house to the zoo.



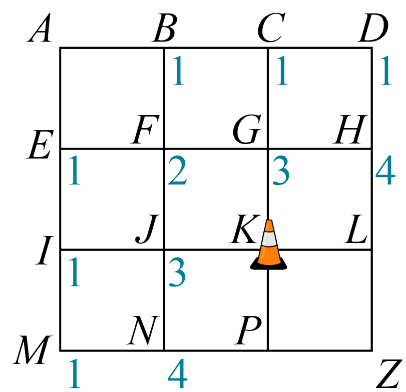
(b) The counting of possible routes follows the same pattern as in (a), except for K and all intersections south and east of it, as shown.



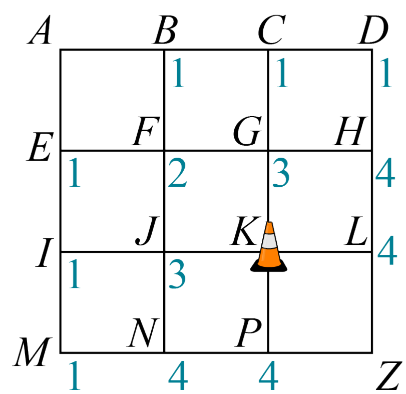
To reach K, Armaan would need to come from J, so there are 3 routes to K. We can then continue counting as before, by adding the number of routes to the preceding intersections. Then we can determine that since there are 7 routes to P and 7 routes to L, there are $7 + 7 = 14$ different routes from Armaan's house to the zoo on Tuesday.



(c) The counting of possible routes follows the same pattern as in (a), except for K and all intersections south and east of it, as shown.



Armaan cannot reach K due to construction. To reach L, Armaan would need to come from H, so there are 4 routes to L. Similarly, to reach P, Armaan would need to come from N, so there are 4 routes to P. Since there are 4 routes to P and 4 routes to L, there are $4 + 4 = 8$ different routes from Armaan's house to the zoo on Friday.

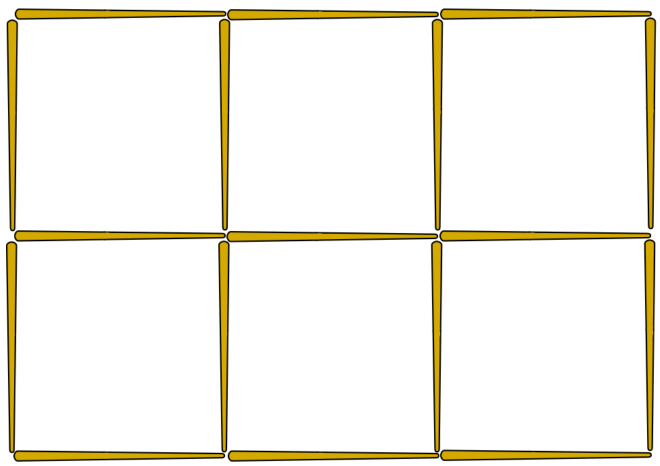


Problem of the Week

Problem B

Toothpicks for Squares

The diagram below is constructed from 17 toothpicks, creating a total of eight squares. Note that some of these are smaller squares of dimension 1 toothpick by 1 toothpick and some are larger squares of dimension 2 toothpicks by 2 toothpicks.



Start with the original diagram in each part below.

(a) Remove five toothpicks so that a total of five squares remain.

(b) Remove five toothpicks so that a total of three squares remain.

(c) Remove three toothpicks so that a total of two squares remain.

(d) Remove six toothpicks so that a total of two squares remain.

Compare your answers to those of a classmate. Are they the same? Can you complete each part without leaving extra toothpicks that do not belong to a square?

Theme: Geometry & Measurement

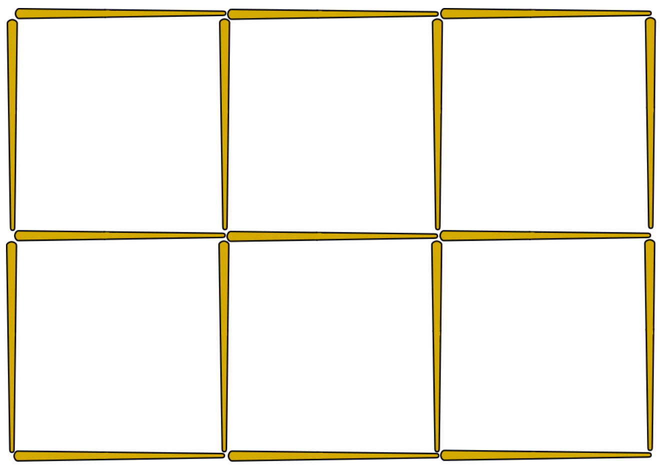
Problem of the Week

Problem B and Solution

Toothpicks for Squares

Problem

The diagram below is constructed from 17 toothpicks, creating a total of eight squares. Note that some of these are smaller squares of dimension 1 toothpick by 1 toothpick and some are larger squares of dimension 2 toothpicks by 2 toothpicks.



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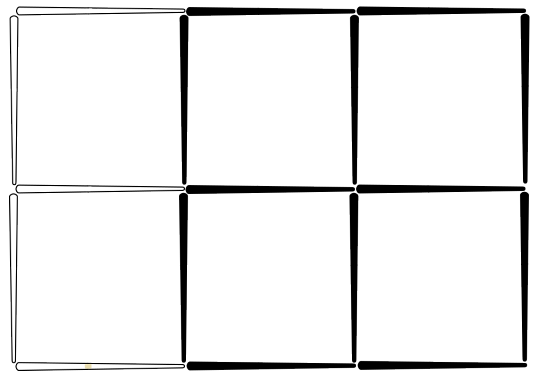
(d) Remove six toothpicks so that a total of two squares remain.

Compare your answers to those of a classmate. Are they the same? Can you complete each part without leaving extra toothpicks that do not belong to a square?

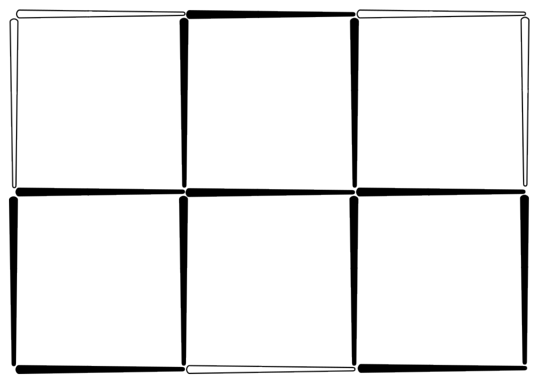
Solution

Answers will vary. The toothpicks removed are coloured white, and the toothpicks remaining coloured black.

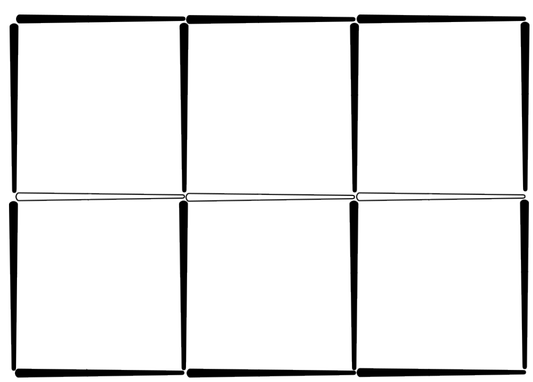
(a) A solution with four small squares and one large square remaining is shown.



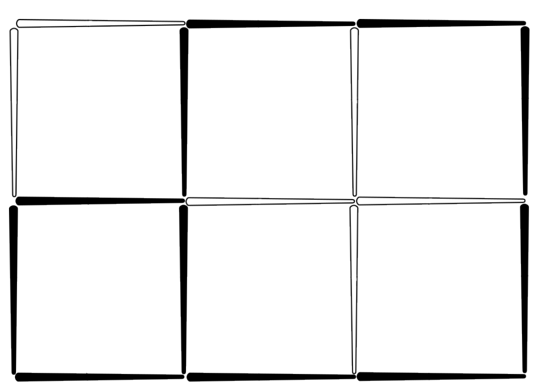
(b) A solution with three small squares remaining is shown.



(c) A solution with two large squares remaining is shown.



(d) A solution with one small and one large square remaining is shown.



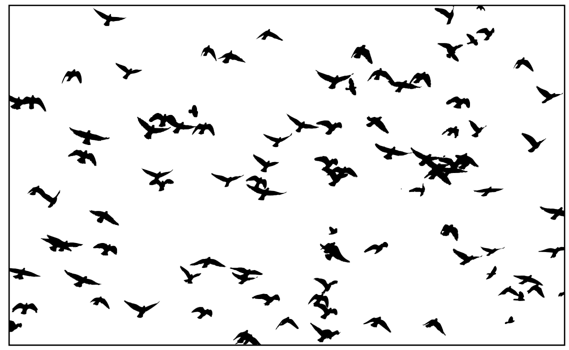
Problem of the Week

Problem B

Destination: Estimation

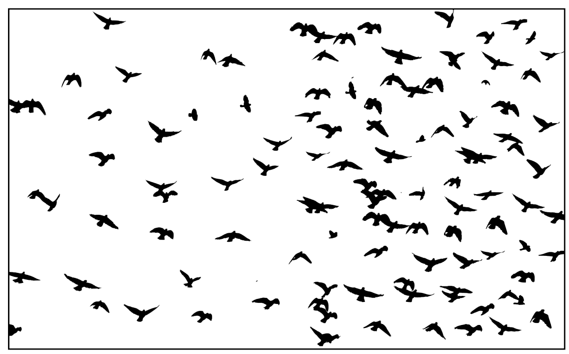
(a) Charley wants to estimate the total number of birds in a picture. Her strategy is to divide the picture into eight smaller rectangles of equal area, count the number of birds in one of the smaller rectangles, and then multiply this number by 8. Using this strategy, estimate the number of birds in Picture 1.

Picture 1



(b) Why might Charley's strategy not give a reliable estimate for the number of birds in Picture 2? Determine a strategy that will give a more reliable estimate, then use your strategy to estimate the number of birds in the picture.

Picture 2



(c) Determine a strategy to estimate the total number of shipping containers on the boat in the picture, then use your strategy to get an estimate. How reliable do you think your estimate is? Explain.

Themes: Geometry & Measurement, Number Sense

Problem of the Week

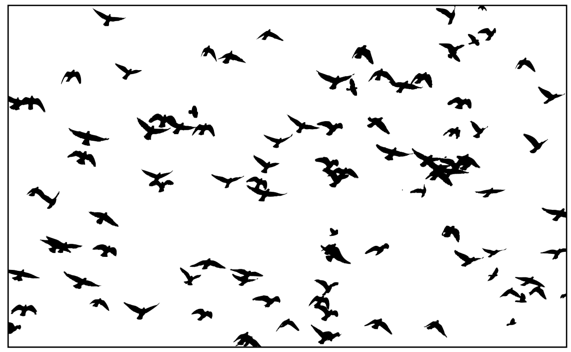
Problem B and Solution

Destination: Estimation

Problem

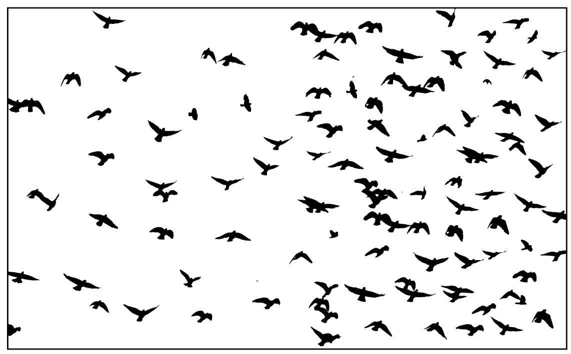
(a) Charley wants to estimate the total number of birds in a picture. Her strategy is to divide the picture into eight smaller rectangles of equal area, count the number of birds in one of the smaller rectangles, and then multiply this number by 8. Using this strategy, estimate the number of birds in Picture 1.

Picture 1



(b) Why might Charley's strategy not give a reliable estimate for the number of birds in Picture 2? Determine a strategy that will give a more reliable estimate, then use your strategy to estimate the number of birds in the picture.

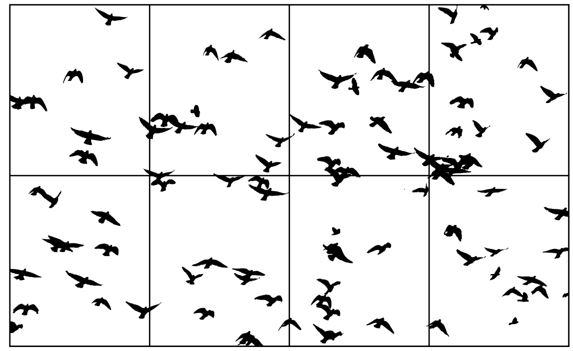
Picture 2



(c) Determine a strategy to estimate the total number of shipping containers on the boat in the picture, then use your strategy to get an estimate. How reliable do you think your estimate is? Explain.

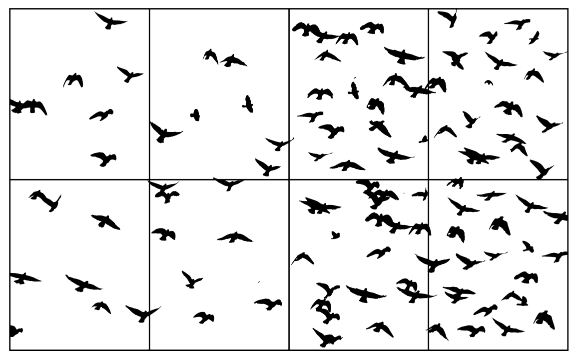
Solution

(a) One way to divide the picture into eight smaller rectangles of equal area is by dividing it into two rows of equal height and four columns of equal width, as shown. We will choose the bottom-left rectangle and count the number of birds in it. Since there are 12 birds, our estimate for the total number of birds in the picture is $12 \times 8 = 96$. Note that estimates will vary depending on which rectangle is chosen and also on how the picture was divided into eight rectangles.



(b) In Picture 2, there are more birds on the right side than the left side. So Charley's strategy could give us an estimate that is considerably larger or smaller than the actual number of birds, depending on which rectangle is chosen.

One strategy is to divide the picture into 8 smaller rectangles like in part (a). The 4 rectangles on the left have few birds, while the 4 rectangles on the right have a lot of birds. We then choose two rectangles; one with few birds and one with a lot of birds, and count the number of birds in each. We then calculate the average (mean) of these and multiply it by the number of rectangles to get an estimate.



In our case, we will choose the top-left and bottom-right rectangles. We counted 7 birds in the top-left rectangle and 22 birds in the bottom-right rectangle. The sum of these is $7 + 22 = 29$ so the average is $29 \div 2 = 14.5$. Then our estimate is $14.5 \times 8 = 116$. Note that estimates will vary depending on the strategy used.

This is more reliable than Charley's strategy because it takes into account the fact that some rectangles have significantly more birds than others.

(c) It is very difficult to determine a reliable estimate for the number of shipping containers because we can't see them all. One strategy is to separate the shipping containers into rows, where each row is parallel to the back of the boat. The only row that we can see clearly is the closest row, where we counted 53 shipping containers. If we assume that the average number of shipping containers in all rows is 53, then we can multiply this by the number of rows to get an estimate. We counted 16 rows, so that means our estimate is $53 \times 16 = 848$. Answers will vary depending on the strategy used, but no strategy can give a very reliable estimate because we can't see so many of the shipping containers.

Problem of the Week

Problem B

Body Math

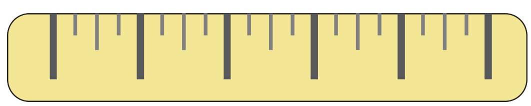
Using wooden rulers, classmates make the following measurements and calculate ratios.

(a) Alara measures the width of her head to be 12 cm. She then measures the width of her eye to be 2.4 cm. What is the ratio of the width of her head to the width of her eye?

(b) Romaisa measures the height of her head to be 20 cm. She then measures her total height to be 1.4 m. What is the ratio of the height of her head to her total height?

(c) Brody measures the length of his nose to be 5 cm. If the ratio of the length of his nose to the length of his index finger is $2:3$, then what is the length of his index finger, in centimetres?

(d) Using wooden rulers, calculate the measurements from parts (a), (b), and (c) yourself. Are your ratios similar?



Theme: Geometry & Measurement

Problem of the Week
Problem B and Solution
Body Math

Problem

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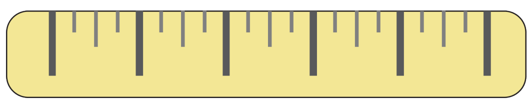


Solution

(a) Since $12 = 5 \times 2.4$, the ratio of the width of her head to the width of her eye is $5:1$.

(b) We first convert 1.4 m to cm. Since there are 100 cm in 1 m, we have 1.4 m = 140 cm.

Since $140 = 7 \times 20$, the ratio of the height of her head to her total height is $1:7$.

(c) The ratio of the length of his nose to the length of his index finger is $2:3$. The ratio $2:3$ means that for every 2 parts of the nose, the index finger has 3 parts. Since 2 parts of the nose is 5 cm, this means 1 part is equal to $5 \div 2 = 2.5$ cm. The index finger corresponds to 3 parts, so we multiply 2.5 by 3. This gives the length of the index finger as $3 \times 2.5 = 7.5$ cm.

Thus, if the length of his nose is 5 cm, the length of his index finger is 7.5 cm.

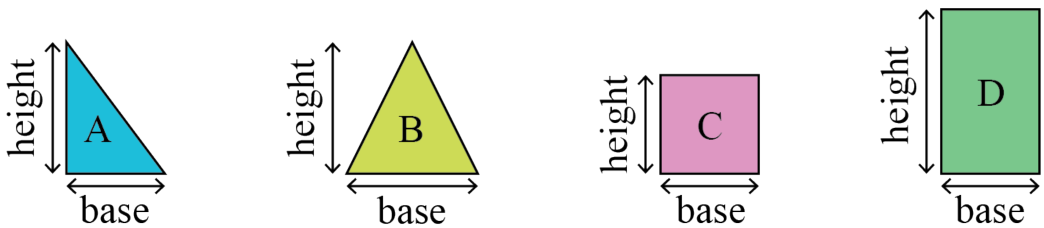
(d) Answers will vary.

Problem of the Week

Problem B

Another Dimension

Kalle draws four shapes on grid paper. Shape A is a right-angled triangle, shape B is an isosceles triangle, shape C is a square, and shape D is a rectangle. Each shape has a horizontal base and a vertical height.



Using the following clues, determine the base and height for each shape.

1. The base of shape A is equal to the base of shape D.
2. The base of shape A is one unit less than the base of shape B.
3. The height of shape C is equal to the base of shape A.
4. The height of shape B, the height of shape A, and the base of shape B are all equal.
5. The area of shape C is 9 square units.
6. The total area of all four shapes is 38 square units.

Themes: Computational Thinking, Geometry & Measurement

Problem of the Week

Problem B and Solution

Another Dimension

Problem

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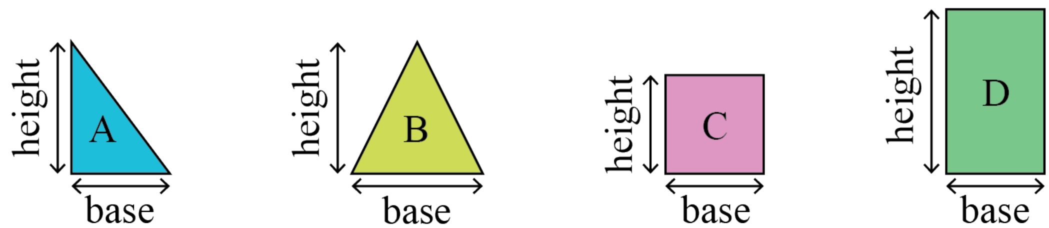


Using the following clues, determine the base and height for each shape.

1. The base of shape A is equal to the base of shape D.
2. The base of shape A is one unit less than the base of shape B.
3. The height of shape C is equal to the base of shape A.
4. The height of shape B, the height of shape A, and the base of shape B are all equal.
5. The area of shape C is 9 square units.
6. The total area of all four shapes is 38 square units.

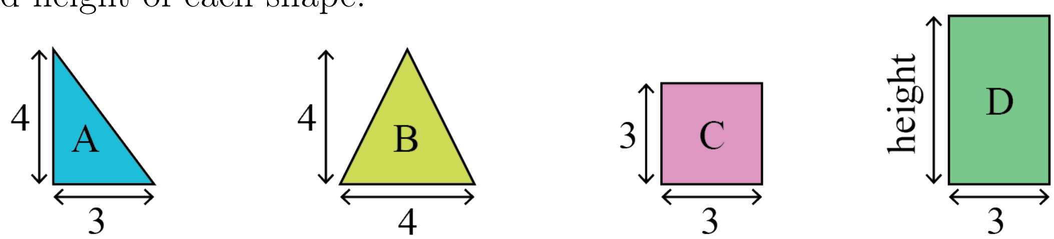
Solution

First we look at clue 5. Since the area of shape C is 9 square units and we know shape C is a square, then it must have a side length of 3 units, since $3 \times 3 = 9$. Thus, the base and height of shape C are each 3 units.

Then from clue 3 we can determine that the base of shape A is 3 units. Then from clue 1 we can determine that the base of shape D is also 3 units.

Then from clue 2 we can determine that the base of shape B must be one unit greater than the base of shape A. Thus, the base of shape B is $3 + 1 = 4$ units.

Then from clue 4 we can determine that the height of shape B and the height of shape A are also 4 units. We now fill in the information we know so far about the base and height of each shape.



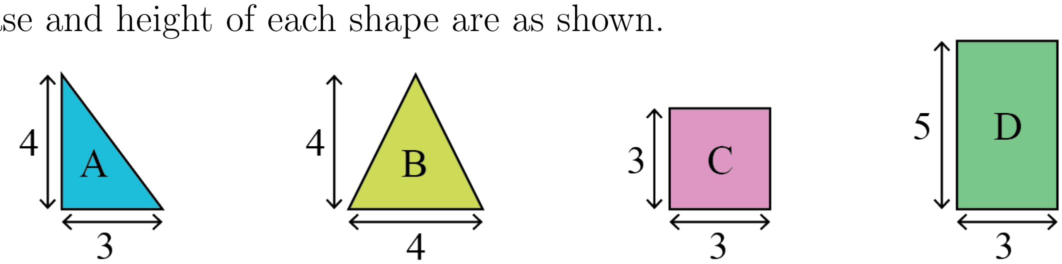
Thus, the only information we still need is the height of shape D. We can determine this using clue 6. First we will calculate the area of each shape.

- Shape A is a triangle, so its area is base $\times$ height $\div 2 = 3 \times 4 \div 2 = 12 \div 2 = 6$ square units.
- Shape B is a triangle, so its area is base $\times$ height $\div 2 = 4 \times 4 \div 2 = 16 \div 2 = 8$ square units.
- Shape C is a square, so its area is base $\times$ height $= 3 \times 3 = 9$ square units.

Thus, the total area of shapes A, B, and C is $6 + 8 + 9 = 23$ square units. Since the total area of all four shapes is 38 square meters, it follows that the area of shape D must be $38 - 23 = 15$ square units.

Shape D is a rectangle, so its area is base $\times$ height $= 3 \times$ height $= 15$ square units. It follows that its height must be 5 units since $3 \times 5 = 15$.

The base and height of each shape are as shown.



Problem of the Week

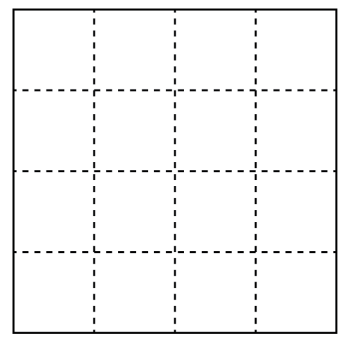
Problem B

Equal Cake and Icing

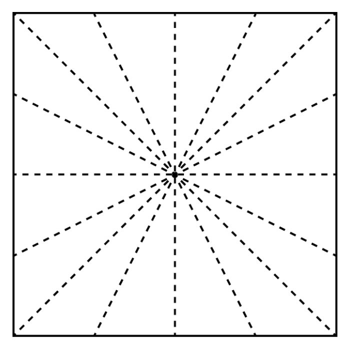
Serenity is having 16 guests for dinner. She baked a cake for dessert using a square cake pan with side length 36 cm. The cake is 8 cm tall. The top face and side faces of the cake are covered in icing.

She would like to slice the cake into 16 pieces. She calls a slicing a "fair cake" if each piece has the same amount (volume) of cake and the same amount (surface area) of icing.

(a) To cut the cake into 16 pieces, suppose she makes three equally-spaced vertical slices and three equally-spaced horizontal slices through the top face of the cake. Is this a fair cake?

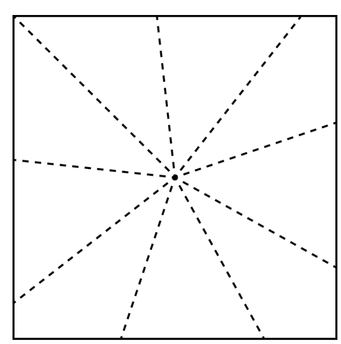


(b) To cut the cake into 16 pieces, suppose she first divides each edge of the top face into four equal lengths. She then makes a straight slice from each end of a length, through the centre of the square, to an end of a length on the opposite edge. Is this a fair cake? Show calculations to support your answer.



Extension:

Only 9 guests want to eat dessert. Serenity decides to cut the cake into 9 pieces by dividing the entire perimeter of the cake into nine equal lengths, starting in the top-left corner and moving clockwise. She then makes a slice from each end of a length to the centre of the square. Is this a fair cake? Show calculations to support your answer.



Theme: Geometry & Measurement

Problem of the Week
Problem B and Solution
Equal Cake and Icing

Problem

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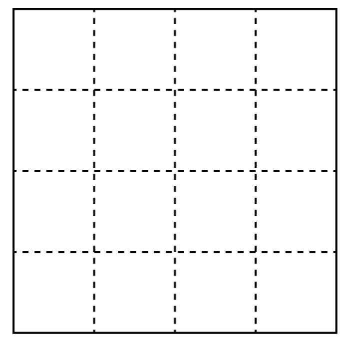

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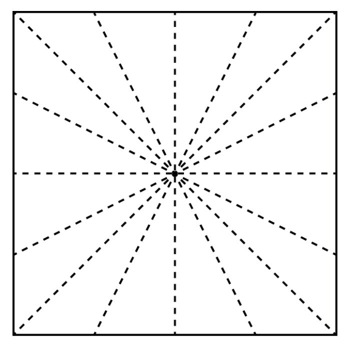

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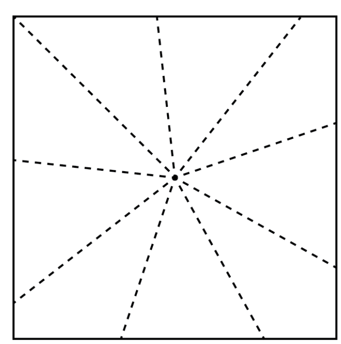


Solution

(a) Since the side length of the square pan is 36 cm, each slice has a square top face with side length $36 \div 4 = 9$ cm.

Since the height of the cake is 8 cm, the volume of each slice is $9 \times 9 \times 8 = 648$ cm^3. Thus, each slice has the same volume of cake.

The top face of each slice has $9 \times 9 = 81$ cm^2 of icing. Each side face with icing will have $9 \times 8 = 72$ cm^2 of icing. Thus, the corner pieces will have $81 + 72 + 72 = 225$ cm^2 of icing, the edge pieces that are not corner pieces will have $81 + 72 = 153$ cm^2 of icing, and the middle pieces will have only 81 cm^2 of icing.

Since each slice does not have the same amount of icing, this is not a fair cake.

(b) The top face of each slice is in the shape of a triangle. Since the side length of the square pan is 36 cm, the base of each triangle is $36 \div 4 = 9$ cm. The height of each triangle is half of the side length of the square pan, or $36 \div 2 = 18$ cm. Using the formula for area of a triangle, we have that the area of the top face of each slice is base $\times$ height $\div 2 = 9 \times 18 \div 2 = 81$ cm^2.

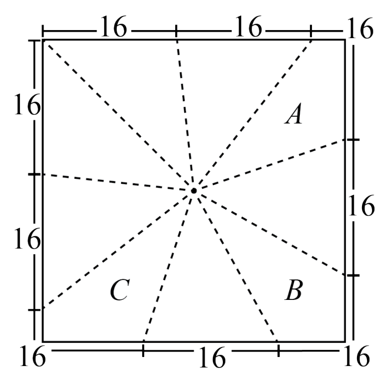
Since each slice has the same top face area of 81 cm^2 and same height of 8 cm, each slice has the same volume of $81 \times 8 = 648$ cm^3.

The top face of each slice has 81 cm^2 of icing. Since the base of each triangular top face is 9 cm and the height of each slice is 8 cm, each slice has a side face with $9 \times 8 = 72$ cm^2 of icing. Thus, each slice has the same amount of icing, $81 + 72 = 153$ cm^2.

Since each slice has the same volume and the same area of icing, this is a fair cake.

Solution to extension:

Since the perimeter of the cake is $36 \times 4 = 144$ cm and 9 slices are made, then each piece will have a total edge length of $\frac{144}{9} = 16$ cm with icing. Thus, since the height of the cake is 8 cm, the amount of icing on the side of each slice is $16 \times 8 = 128$ cm^2.



For six of the slices, the top face of the slice is a triangle. For the remaining three slices, the top face is a quadrilateral. These slices are marked A, B, and C.

First we look at the triangular slices. The base of each triangle is 16 cm and the height is half the side length of the square pan, or $36 \div 2 = 18$ cm. Using the formula for area of a triangle, we have that the area of the top face of each triangular slice of cake is base $\times$ height $\div 2 = 16 \times 18 \div 2 = 144$ cm^2.

Next we look at the quadrilaterals. Each quadrilateral consists of two triangles, each with height 18 cm.

The top face of slice A has one triangle with base length $36 - 16 - 16 = 4$ cm. Thus, the other triangle has base length equal to $16 - 4 = 12$ cm. Therefore, using the formula for the area of a triangle, we can determine that the area of the top face of slice A is $4 \times 18 \div 2 + 12 \times 18 \div 2 = 36 + 108 = 144$ cm^2.

The top face of slice B has one triangle with base length $36 - 12 - 16 = 8$ cm. Thus, the other triangle has base length equal to $16 - 8 = 8$ cm. Therefore, using the formula for the area of a triangle, we can determine that the area of the top face of slice B is $8 \times 18 \div 2 + 8 \times 18 \div 2 = 72 + 72 = 144$ cm^2.

The top face of slice C has one triangle with base length $36 - 8 - 16 = 12$ cm. Thus, the other triangle has base length equal to $16 - 12 = 4$ cm. Since these are equal to the base lengths of the triangles in the top face of slice A, it follows that the area of the top face of slice C is also 144 cm^2.

Therefore, since each slice has the same top face area of 144 cm^2 and the same height of 8 cm, each slice has volume equal to $144 \times 8 = 1152$ cm^2.

Also, each slice has 144 cm^2 of icing on top and 128 cm^2 of icing on the side, for a total of $144 + 128 = 272$ cm^2 of icing.

Since each slice has the same volume and the same area of icing, this is a fair cake.

Number Sense (N)

Take me to the cover

Problem of the Week

Problem B

Letters with Friends

In the game "Scribble", players place tiles containing individual letters on a board to form words. The board is divided up into squares, and each letter in a word is placed in adjacent squares in a row (reading from left to right) or a column (reading from the top down). Each letter has a point value, and the score for a word is found by adding up the point values for each of its letters. The point values for each letter are shown in the following table.

Letter	Number of Points
A, E, I, L, N, O, R, S, T	1
D, G, U, Y	2
B, C, M, P, W	3
F, H	5
J, K, V	8
Q, X, Z	10

For example, the word POUR would have a score of $3 + 1 + 2 + 1 = 7$.

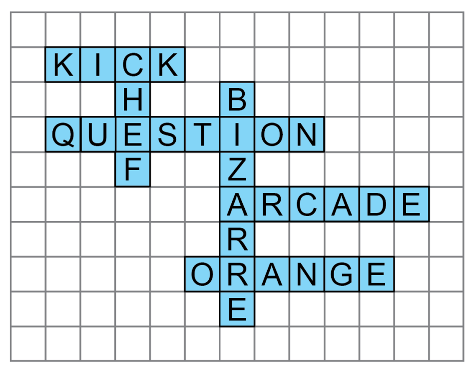
Players alternate turns making words on the board. After the first turn, each new word must build off of one of the letters already on the board. The score for any new word includes the point value for the letter that was already on the board.

Vinny and Theo played Scribble. Their game board is shown after they each had three turns.

(a) Which word on the game board has the highest score?

(b) Which word on the game board has the lowest score?

(c) If Vinny made the first word, what is the maximum total score that he could have for his three words? In order to achieve this score, which word(s) could he have started with?



Extension: Suppose Vinny's next word has a score of 10. Which word could this be? See how many different possible words you can find. You can assume that the game board extends past the size of what is shown above.

Theme: Number Sense

Problem of the Week

Problem B and Solution

Letters with Friends

Problem

In the game "Scribble", players place tiles containing individual letters on a board to form words. The board is divided up into squares, and each letter in a word is placed in adjacent squares in a row (reading from left to right) or a column (reading from the top down). Each letter has a point value, and the score for a word is found by adding up the point values for each of its letters. The point values for each letter are shown in the following table.

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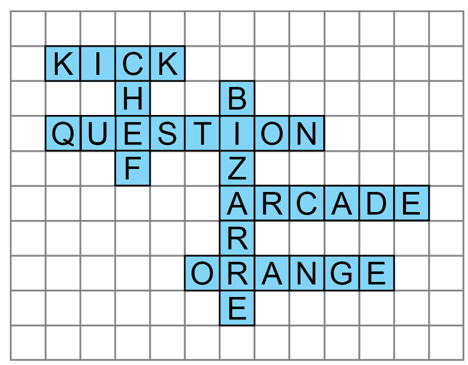


Extension: Suppose Vinny's next word has a score of 10. Which word could this be? See how many different possible words you can find. You can assume that the game board extends past the size of what is shown above.

Solution

(a) The word KICK has the highest score. Its score is $8 + 1 + 3 + 8 = 20$.

(b) The word ORANGE has the lowest score. Its score is $1 + 1 + 1 + 1 + 2 + 1 = 7$.

(c) The three words with the highest score are KICK, QUESTION, and BIZARRE. Their scores are 20, 18, and 18, respectively. If Vinny made all three of these words, then his total score would be $20 + 18 + 18 = 56$.

It is possible for Vinny to place all three of these words. He could have started with QUESTION or BIZARRE, as shown in the tables below.

Turn	Word
Vinny's first turn	QUESTION
Theo's first turn	CHEF
Vinny's second turn	BIZARRE
Theo's second turn	ARCADE
Vinny's third turn	KICK
Theo's third turn	ORANGE

Turn	Word
Vinny's first turn	BIZARRE
Theo's first turn	ARCADE
Vinny's second turn	QUESTION
Theo's second turn	CHEF
Vinny's third turn	KICK
Theo's third turn	ORANGE

However, if Vinny started with KICK, then Theo would have made CHEF next, and then Vinny would have made QUESTION. The only possible next word is BIZARRE, so Theo must have made BIZARRE. Thus, if Vinny started with KICK, then he could not have achieved the maximum total score of 56. Therefore, Vinny could have started with QUESTION or BIZARRE.

Solution to Extension:

There are many possible words. It helps by looking at the game board to find the letters that Vinny could build off of. Here are some possible words he could make that have a score of 10.

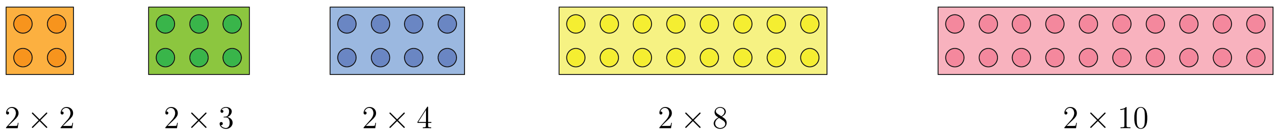
- Using one of the Ks from KICK he could make ASK.
- Using the N from QUESTION he could make CHIN.
- Using the E from ARCADE he could make SIMPLE.
- Using the N from ORANGE he could make NIGHT.
- Using the G from ORANGE he could make GIFTS.
- Using the E from ORANGE he could make EACH.

Problem of the Week

Problem B

Playing with Bricks

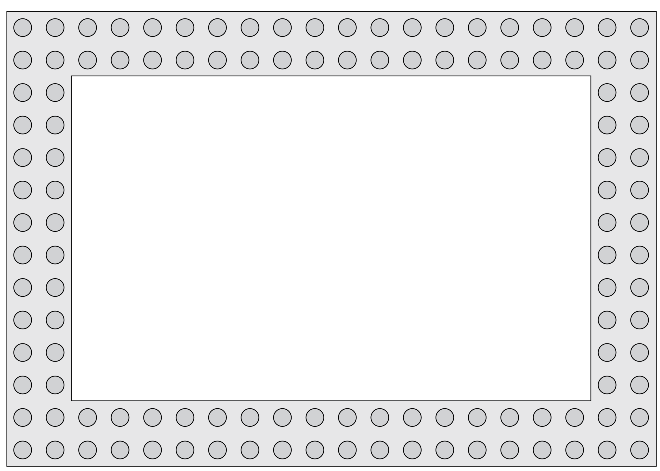
Saskia has the following five sizes of Lego™ bricks.



For each question below, assume that Saskia will never run out of bricks.

(a) Saskia wants to make a row of bricks that measures 2×16. Which of her brick sizes can she use if all bricks used must be the same size?

(b) Saskia wants to make a rectangular frame of bricks that measures 14×20 on the outside, as shown. Which of her brick sizes can she use if all bricks used must be the same size?



(c) Saskia wants to make a rectangular frame of bricks that measures 320×420 on the outside. What is the largest brick size that she can use if all bricks used must be the same size?

Themes: Geometry & Measurement, Number Sense

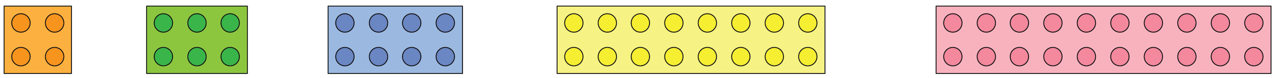
Problem of the Week

Problem B and Solution

Playing with Bricks

Problem

Saskia has the following five sizes of Lego™ bricks.



2×2 2×3 2×4 2×8 2×10

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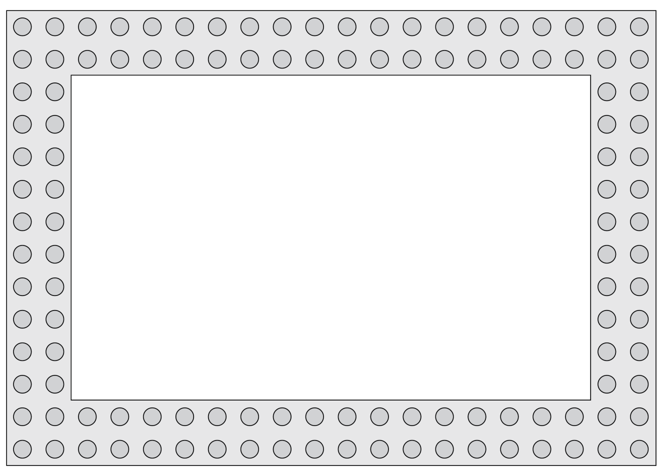


(c) Saskia wants to make a rectangular frame of bricks that measures 320×420 on the outside. What is the largest brick size that she can use if all bricks used must be the same size?

Solution

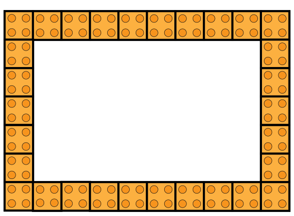
In our solutions, we call the smaller number in the brick size the *width*, and call the larger number the *length*. So a 2×4 brick has a width of 2 and a length of 4.

(a) Saskia can make a row that measures 2×16 by using eight 2×2 bricks, four 2×4 bricks, or two 2×8 bricks.

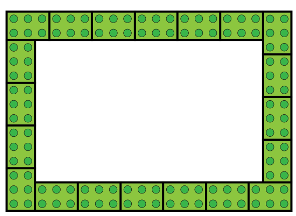
In general, since the width of the row is the same as the width of each brick, we can focus only on the lengths. If we want to place some number of the same brick in a row and have the total length equal 16, then the possible bricks are ones whose length is a factor of 16. These are 2×2, 2×4, and 2×8.

(b) We can think of the rectangular frame as two 2×20 rows on the top and bottom, and two 2×10 rows on the sides. If we placed these rows end to end, they would create a row with length $20 + 20 + 10 + 10 = 60$. Since 8 is the only length that is not a factor of 60, we know for sure that it is not possible to make the frame using only 2×8 bricks.

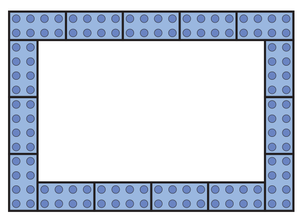
For the other brick sizes, we need to show that it is possible to place them around the frame. For each brick size, we start at the top-left corner and place bricks horizontally moving to the right. Once we can no longer place bricks horizontally, we start placing the bricks vertically, moving down. We continue in this clockwise fashion, until all bricks are placed around the frame. This method works for the 2×2, 2×3, 2×4, and 2×10 bricks, as shown.



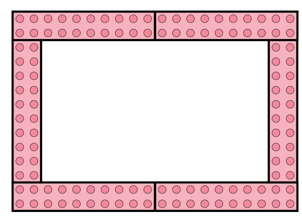
2×2



2×3



2×4



2×10

Thus, the possible brick sizes are 2×2, 2×3, 2×4, and 2×10.

(c) We can think of the rectangular frame as two 2×320 rows on the sides, and two 2×416 rows on the top and bottom. If we placed these rows end to end, they would create a row with length $320 + 320 + 416 + 416 = 1472$.

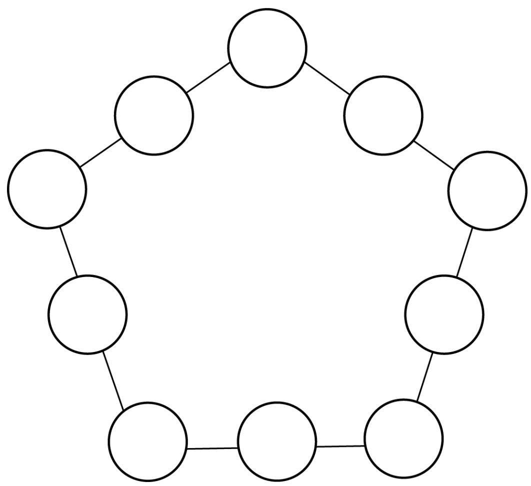
Since 10 is not a factor of 1472, we know it is not possible to make the frame using only 2×10 bricks. However, 8 is a factor of 1472, so we will attempt to place the 2×8 bricks around the frame. It helps to notice that 8 is a factor of both 320 and 416. Thus, we can make two 2×320 rows and two 2×416 rows out of 2×8 bricks. Putting these four rows together, with the 2×320 rows on the sides, and the 2×416 rows on the top and bottom gives the desired frame. Thus, the largest brick size that Saskia can use is 2×8.

Problem of the Week

Problem B

A Pentagon Puzzle

To solve a puzzle, Jonah needs to insert a different number from 1 to 10 into each of the the ten circles in the diagram so that the three numbers on each side of the pentagon have the same sum. He calls this sum the *magic sum*.



(a) What is the least possible sum of any three of the given digits? What is the greatest possible sum of any three of the given digits?

(b) Do you think either of the sums in part (a) could be the magic sum? Explain.

(c) Find a solution to the puzzle.

EXTENSION: Can you find a different solution that is not a rotation or reflection of your solution from (c)?

THEME: NUMBER SENSE

Problem of the Week
Problem B and Solution
A Pentagon Puzzle

Problem

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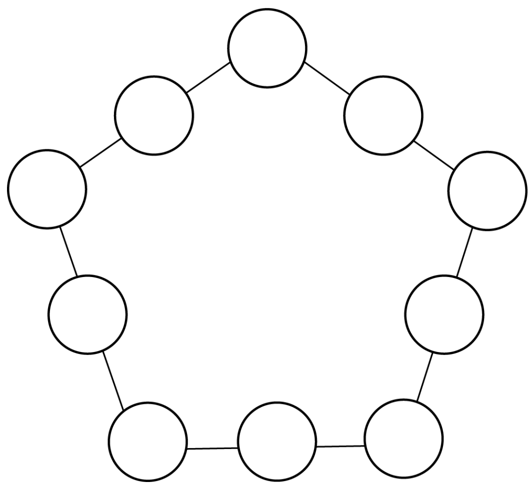

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(b) Do you think either of the sums in part (a) could be the magic sum? Explain.

(c) Find a solution to the puzzle.

Extension: Can you find a different solution that is not a rotation or reflection of your solution from (c)?

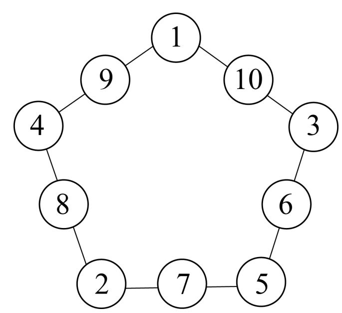
Solution

(a) The least possible sum is $1 + 2 + 3 = 6$. The greatest possible sum is $8 + 9 + 10 = 27$.

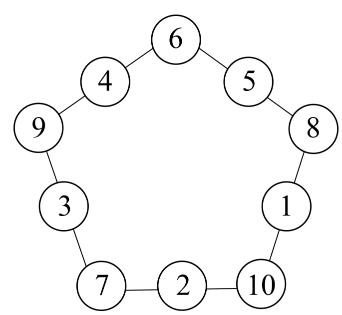
(b) The magic sum cannot be 6, since all other possible sums of three numbers are greater than 6. Similarly, the magic sum cannot be 27, since all other possible sums are less than 27.

(c) When solving this problem, it helps to know what the magic sum is. However, the magic sum will not be the same for every solution, because it depends on where the numbers are placed. For a particular arrangement, if we add up the sum of the numbers on each side of the pentagon, we will have counted the numbers in the corners twice. So the magic sum equals the sum of all the numbers, plus the sum of just the numbers in the corners, divided by five.

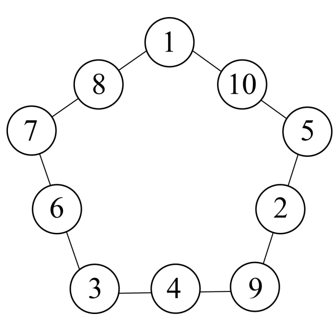
We can start by placing the smallest numbers, 1, 2, 3, 4, and 5, in the corners. The sum of all the numbers is $1 + 2 + \cdots + 9 + 10 = 55$. The sum of the numbers in the corners is $1+2+3+4+5 = 15$, and $55+15 = 70$. Then, the magic sum is $70 \div 5 = 14$. This is the smallest possible magic sum because the smallest numbers are in the corners. A solution with a magic sum of 14 is shown.

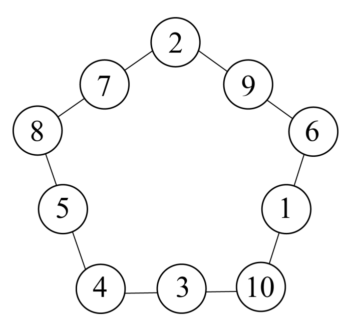


To find the largest possible magic sum, we will place the largest numbers, 6, 7, 8, 9, and 10, in the corners. The sum of the numbers in the corners is $6 + 7 + 8 + 9 + 10 = 40$, and $55 + 40 = 95$. Then, the magic sum is $95 \div 5 = 19$. A solution with a magic sum of 19 is shown.



Two other solutions, with magic sums of 16 and 17, are shown.





Problem of the Week
Problem B
Baking Cookies

Felix and Vera are baking cookies. Their recipe bakes 12 cookies and uses the following ingredients:

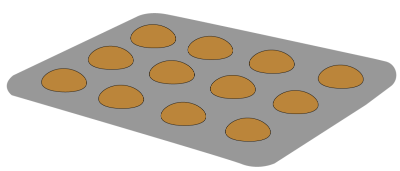
- $\frac{1}{2}$ cup of butter
- $\frac{1}{3}$ cup of sugar
- 1 cup of flour
- $\frac{2}{3}$ teaspoon of vanilla

(a) Felix and Vera decide to triple the recipe. How much of each ingredient will they need?

(b) The cookies are so good that Felix and Vera plan to make 60 cookies for a fundraiser.

(i) How much butter will they need?

(ii) Each batch of cookies takes 11 minutes to bake, and their oven can fit only 24 cookies at a time. How long will it take to bake all the cookies for the fundraiser?



Theme: Number Sense

Problem of the Week

Problem B and Solution

Baking Cookies

Problem

Felix and Vera are baking cookies. Their recipe bakes 12 cookies and uses the following ingredients:

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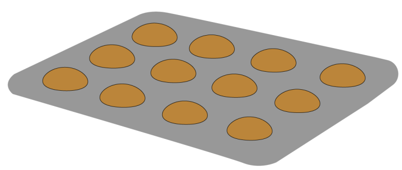

Solution

(a) If they triple the recipe, then the amounts of each ingredient will be as follows:

- Butter: $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$ cups
- Sugar: $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ cup
- Flour: $1 + 1 + 1 = 3$ cups
- Vanilla: $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{6}{3} = 2$ teaspoons

(b) (i) Since $12 \times 5 = 60$, they will need to make 5 batches of cookies. So the amount of butter they will need is $5 \times \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$ cups.

(ii) Their oven can fit only 24 cookies at a time, which is the same as 2 batches. Since they need to make 5 batches, they will need to bake 2 batches, then another 2 batches, then 1 batch. So the total baking time would be $11 + 11 + 11 = 33$ minutes.

Problem of the Week
Problem B
Time to Vote

Every four years, Bigland will vote for a prime minister. In 2022, Bigland had a population of 34 million. Of that number, 72% were eligible to vote, but 60% of the population that was eligible to vote actually voted.

(a) How many people in Bigland actually voted in 2022? What percentage and fraction of the population of Bigland does this represent?

(b) For Tinyland, there were two million, seven hundred thirty-six thousand, six hundred twenty-eight people eligible to vote in 2022. If two million, thirty-four thousand, three hundred twenty-eight people actually voted, how does this voter turnout compare to the 60% voter turnout of Bigland?

Theme: Number Sense

Problem of the Week

Problem B and Solution

Time to Vote

Problem

Every four years, Bigland will vote for a prime minister. In 2022, Bigland had a population of 34 million. Of that number, 72% were eligible to vote, but 60% of the population that was eligible to vote actually voted.

(a) How many people in Bigland actually voted in 2022? What percentage and fraction of the population of Bigland does this represent?

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Solution

(a) The number of eligible voters in Bigland is 72% of 34 million people, or $0.72 \times 34\,000\,000 = 24\,480\,000$ people.
Of these, the number who actually voted is $0.60 \times 24\,480\,000 = 14\,688\,000$ people.
This represents the fraction $\frac{14\,688\,000}{340\,000\,000} = \frac{14\,688}{34\,000} = \frac{7344}{17\,000} = \frac{54}{125}$, which is equivalent to the decimal 0.432, or 43.2% of the population of Bigland.

Alternatively, 60% of 72% is $0.62 \times 0.7 = 0.432$ or 43.2%, which can be expressed as $\frac{432}{1000} = \frac{54}{125}$.

(b) In Tinyland, $2\,736\,628$ people were eligible to vote, of which $2\,034\,328$ people actually voted. Thus, as a fraction, the voter turnout was $\frac{2\,034\,328}{2\,736\,628} \approx 0.743$, or 74.3%, significantly higher than the 60% voter turnout in Bigland.

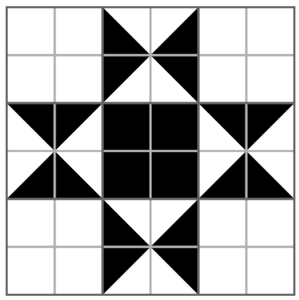
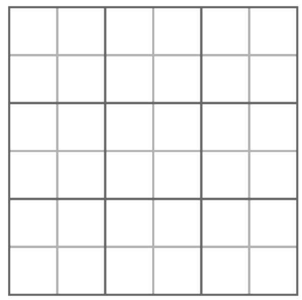
Problem of the Week

Problem B

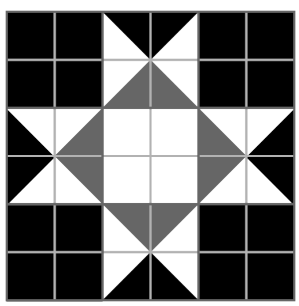
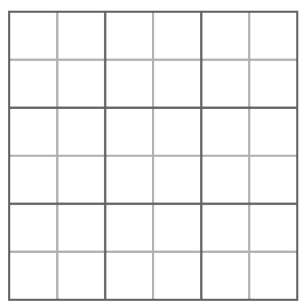
Quilt Designs for Barns

Sometimes people decorate their barns with quilts. These barn quilts are often designed in a square grid with a variety of colours and patterns, and they usually have symmetry in such a way that if they're rotated they will look the same. In the 6 by 6 grids below, you are going to explore and create some barn quilt designs. In each part, you can add straight lines as needed.

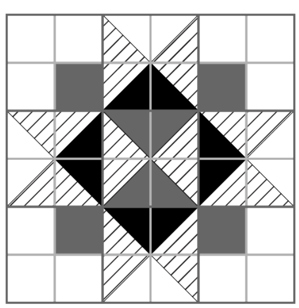
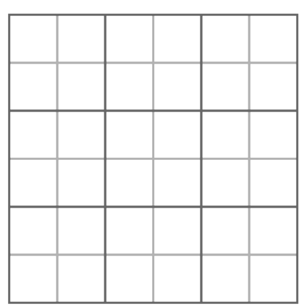
(a) What fraction of the barn quilt design below is black, and what fraction is white? In the blank grid, design another barn quilt using two colours and the same fractions.

(b) What fraction of the barn quilt design below is black? Grey? White? In the blank grid, design another barn quilt using three colours and the same fractions.

(c) What fraction of the following barn quilt design below is black? Grey? Striped? White? In the blank grid, design another barn quilt using four colours and the same fractions.

Themes: Geometry & Measurement, Number Sense

Problem of the Week

Problem B and Solution

Quilt Designs for Barns

Problem

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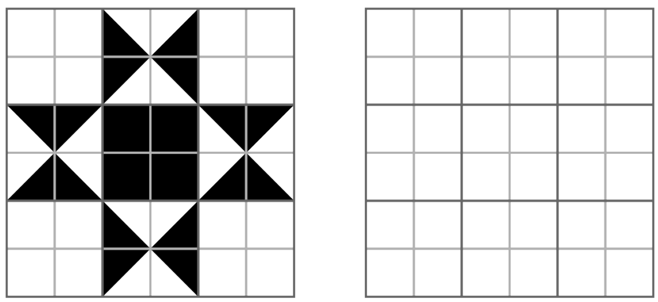
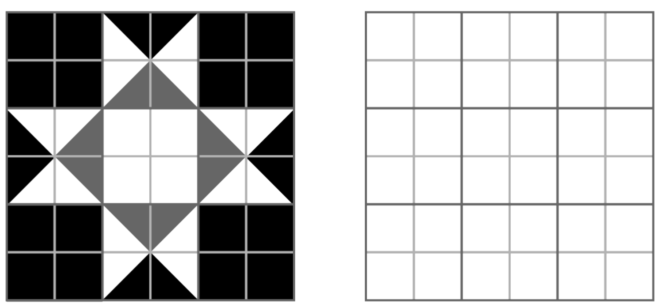
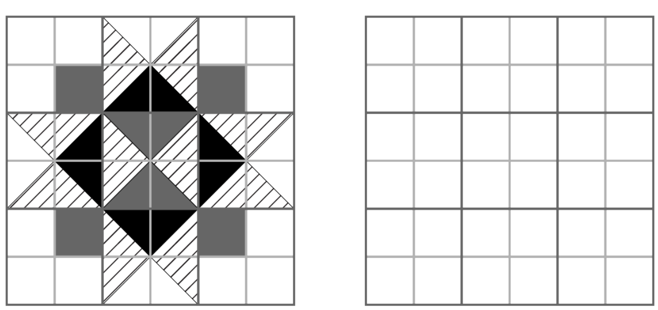


(b) What fraction of the barn quilt design below is black? Grey? White? In the blank grid, design another barn quilt using three colours and the same fractions.



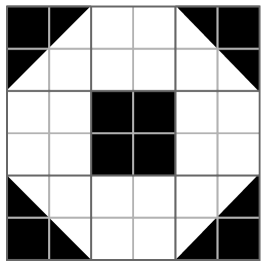
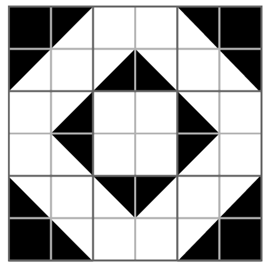
(c) What fraction of the following barn quilt design below is black? Grey? Striped? White? In the blank grid, design another barn quilt using four colours and the same fractions.



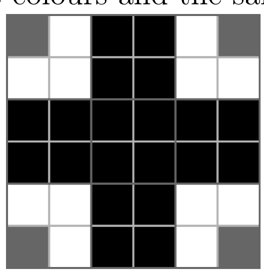
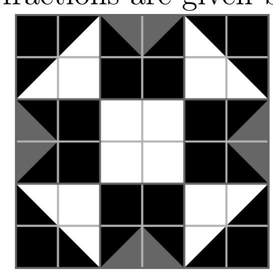
Solution

Note that a 6 by 6 grid consists of 36 smaller squares. There are also smaller triangles in the designs which each make up half of a smaller square. (These smaller triangles are formed when the two diagonals of a 2 by 2 square in the grid are drawn.)

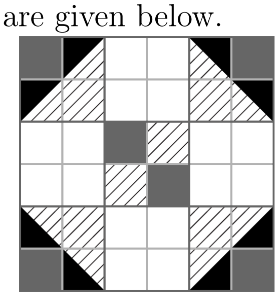
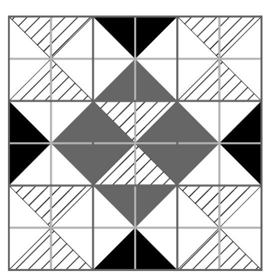
(a) The given barn quilt design has 16 white smaller squares plus 16 white smaller triangles, equivalent to $16 \div 2 = 8$ smaller squares. Therefore, the amount that is white is equivalent to $16 + 8 = 24$ smaller squares, and the fraction that is white is $\frac{24}{36} = \frac{2}{3}$. Thus, the fraction that is black must be $1 - \frac{2}{3} = \frac{1}{3}$. Two barn quilt designs using two colours and the same fractions are given below.

(b) The given barn quilt design has 16 black smaller squares plus 8 black smaller triangles, equivalent to $8 \div 2 = 4$ smaller squares. Therefore, the amount that is black is equivalent to $16 + 4 = 20$ smaller squares, and the fraction that is black is $\frac{20}{36} = \frac{5}{9}$. There are 8 grey smaller triangles, equivalent to a total of $8 \div 2 = 4$ grey smaller squares. Therefore, the fraction that is grey is $\frac{4}{36} = \frac{1}{9}$ and the fraction of white is $1 - \frac{5}{9} - \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$. Two barn quilt designs using three colours and the same fractions are given below.

(c) The given barn quilt design has 12 white smaller squares plus 8 white smaller triangles, equivalent to $8 \div 2 = 4$ smaller squares. Therefore, the amount that is white is equivalent to $12 + 4 = 16$ white smaller squares, and the fraction that is white is $\frac{16}{36} = \frac{4}{9}$. There are 8 black smaller triangles, equivalent to a total of $8 \div 2 = 4$ black smaller squares. Therefore, the fraction that is black is $\frac{4}{36} = \frac{1}{9}$. There are 4 smaller grey squares plus 4 grey smaller triangles, equivalent to $4 \div 2 = 2$ smaller squares. Therefore, the amount that is grey is equivalent to $4 + 2 = 6$ grey smaller squares, and the fraction that is grey is $\frac{6}{36} = \frac{1}{6}$. There are 20 striped smaller triangles, equivalent to $20 \div 2 = 10$ striped smaller squares. Therefore, the fraction that is striped is $\frac{10}{36} = \frac{5}{18}$. Two barn quilt designs using four colours and the same fractions are given below.

Problem of the Week
Problem B
The Apartments are Multiplying

Alisa's apartment building has 12 floors, with 6 apartments on each floor. We have the following information about three other apartment buildings.

- Compared to Alisa's building, Bettina's apartment building has the same number of floors, but three times as many apartments on each floor.
- Compared to Bettina's building, Colin's apartment building has twice as many floors, but half as many apartments on each floor.
- Compared to Colin's building, Dara's apartment building has twice as many floors, and three times as many apartments on each floor.

Use this information to complete the following table. How does the total number of apartments change from one row to the next?

Building Owner	Number of Floors	Number of Apartments per Floor	Total Number of Apartments
Alisa	12	6	
Bettina			
Colin			
Dara			

EXTENSION: Ferid's apartment building has 20 apartments in total. Compared to Ferid's building, Gauri's apartment building has five times as many floors, but the same number of apartments per floor. How many apartments in total does Gauri's building have?

THEMES: COMPUTATIONAL THINKING, NUMBER SENSE

Problem of the Week
Problem B and Solution
The Apartments are Multiplying

Problem

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EXTENSION: Ferid's apartment building has 20 apartments in total. Compared to Ferid's building, Gauri's apartment building has five times as many floors, but the same number of apartments per floor. How many apartments in total does Gauri's building have?

Solution

The completed table is shown.

Building Owner	Number of Floors	Number of Apartments per Floor	Total Number of Apartments
Alisa	12	6	72
Bettina	12	18	216
Colin	24	9	216
Dara	48	27	1296

We notice the following about the total number of apartments.

- Bettina's apartment building has 3 times as many apartments as Alisa's. This is because the number of floors did not change but the number of apartments per floor was multiplied by 3, so the total number of apartments was multiplied by 3.

- Colin's apartment building and Bettina's apartment building have the same number of apartments. This is because the number of floors was multiplied by 2, but the number of apartments per floor was divided by 2. Since these are opposite operations, the total number of apartments did not change.
- Dara's apartment building has 6 times as many apartments as Colin's. This is because the number of floors was multiplied by 2, and the number of apartments per floor was multiplied by 3, so the total number of apartments was multiplied by $3 \times 2 = 6$.

Extension Solution:

Since Gauri's apartment building has 5 times as many floors, but the same number of apartments per floor as Ferid's, then the number of apartments in Gauri's building will be equal to 5 times the number of apartments in Ferid's building, which is $20 \times 5 = 100$.

Problem of the Week

Problem B

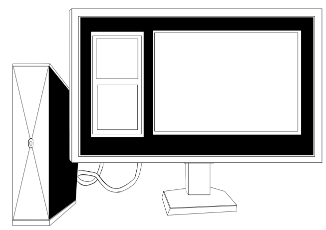
A Taxing Question

Sales tax is added to the price of most items or services that you purchase in Canada. The final price is calculated by finding the amount of sales tax, and adding this amount to original price.

However, the percentage of sales tax is not the same in all provinces.

(a) The latest gaming console, the XStation PlayBox, sells for \$650. What would be the final price for this console in each of the provinces listed in the table below?

Province	Sales Tax	Final Price
Nova Scotia	15%	
Ontario	13%	
Saskatchewan	11%	
Alberta	5%	
British Columbia	12%	



(b) Of the sales tax collected, 5% goes to the federal government, and the rest goes to the province. How much sales tax would the federal and provincial governments get for a console sale in Nova Scotia? What about in Alberta?

EXTENSION: Suppose that you live in British Columbia, but only 50 km from the nearest store in Alberta that sells the console. The price of gas is \$1.50 per litre, and your car uses 10 L of gas per 100 km of driving. Is it cheaper to drive to Alberta to buy the console instead of buying it in British Columbia?

THEME: NUMBER SENSE

Problem of the Week

Problem B and Solution

A Taxing Question

Problem

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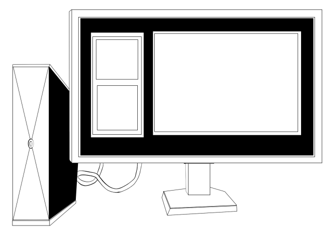
Saskatchewan	11%	
Alberta	5%	
British Columbia	12%	



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Extension: Suppose that you live in British Columbia, but only 50 km from the nearest store in Alberta that sells the console. The price of gas is \$1.50 per litre, and your car uses 10 L of gas per 100 km of driving. Is it cheaper to drive to Alberta to buy the console instead of buying it in British Columbia?

Solution

(a) **Solution 1:** We calculate the amount of sales tax using fractions.

$$\begin{aligned}\text{Amount of sales tax in Nova Scotia} &= 15\% \text{ of } \$650\\ &= 10\% \text{ of } \$650 + 5\% \text{ of } \$650\\ &= \tfrac{1}{10} \times \$650 + 5 \times \tfrac{1}{100} \times \$650\\ &= \$65 + 5 \times \$6.50\\ &= \$97.50\end{aligned}$$

Thus, the final price in Nova Scotia is $\$650 + \$97.50 = \$747.50$.

$$\begin{aligned}\text{Amount of sales tax in Ontario} &= 13\% \text{ of } \$650\\ &= 10\% \text{ of } \$650 + 3\% \text{ of } \$650\\ &= \tfrac{1}{10} \times \$650 + 3 \times \tfrac{1}{100} \times \$650\\ &= \$65 + 3 \times \$6.50\\ &= \$84.50\end{aligned}$$

Thus, the final price in Ontario is $\$650 + \$84.50 = \$734.50$.

$$\begin{aligned}\text{Amount of sales tax in Saskatchewan} &= 11\% \text{ of } \$650\\ &= 10\% \text{ of } \$650 + 1\% \text{ of } \$650\\ &= \tfrac{1}{10} \times \$650 + \tfrac{1}{100} \times \$650\\ &= \$65 + \$6.50\\ &= \$71.50\end{aligned}$$

Thus, the final price in Saskatchewan is $\$650 + \$71.50 = \$721.50$.

$$\begin{aligned}\text{Amount of sales tax in Alberta} &= 5\% \text{ of } \$650\\ &= 5 \times \tfrac{1}{100} \times \$650\\ &= 5 \times \$6.50\\ &= \$32.50\end{aligned}$$

Thus, the final price in Alberta is $\$650 + \$32.50 = \$682.50$.

$$\begin{aligned}\text{Amount of sales tax in British Columbia} &= 12\% \text{ of } \$650\\ &= 10\% \text{ of } \$650 + 2\% \text{ of } \$650\\ &= \tfrac{1}{10} \times \$650 + 2 \times \tfrac{1}{100} \times \$650\\ &= \$65 + 2 \times \$6.50\\ &= \$78.00\end{aligned}$$

Thus, the final price in British Columbia is $\$650 + \$78.00 = \$728.00$.

Solution 2: We calculate the amount of sales tax using decimals.

- Nova Scotia: $0.15 \times \$650 = \$97.50$
  Thus, the final price in Nova Scotia is $\$650 + \$97.50 = \$747.50$.
- Ontario: $0.13 \times \$650 = \$84.50$
  Thus, the final price in Ontario is $\$650 + \$84.50 = \$734.50$.
- Saskatchewan: $0.11 \times \$650 = \$71.50$
  Thus, the final price in Saskatchewan is $\$650 + \$71.50 = \$721.50$.
- Alberta: $0.05 \times \$650 = \$32.50$
  Thus, the final price in Alberta is $\$650 + \$32.50 = \$682.50$.
- British Columbia: $0.12 \times \$650 = \$78.00$
  Thus, the final price in British Columbia is $\$650 + \$78.00 = \$728.00$.

(b) In Nova Scotia, the federal government would get 5% of \$650, which is $0.05 \times \$650 = \$32.50$, while the provincial government would get 10% of \$650, which is $0.10 \times \$650 = \$65.00$.

In Alberta, the federal government would get \$32.50, while the provincial government would get \$0.00.

Extension Solution: The total distance from your house to the store in Alberta and back home is 50 km + 50 km = 100 km. This trip will use 10 L of gas. Since the price of gas is \$1.50 per litre, the total cost of 10 L of gas would be $10 \times \$1.50 = \$15$. Therefore, driving to the store in Alberta and back adds an extra \$15. So the total amount of money spent would be $\$682.50 + \$15 = \$697.50$. Since $\$697.50 < \$728.00$, it is cheaper to drive to Alberta to buy the console than to buy it in British Columbia.

Problem of the Week
Problem B
Does This All Add Up?

In the table shown, the top row and leftmost column are grey, and the remaining numbers form a 3×3 array. Each number in the array is equal to the sum of the numbers in grey in its row and column.

	5	2	7
3	8	5	10
4	9	6	11
6	11	8	13

(a) Follow the steps below.

1. Circle any number in the array (*for example,* 6).
2. Cross off the other numbers in the same row and column of the array. (*For our example, we would then cross out* 9, 11, 5, *and* 8.)
3. Circle any remaining number in the array (*for example,* 13).
4. Cross off the other numbers in the same row and column of the array. (*For our example, we would then cross out* 10 *and* 11.)
5. Circle the remaining number in the array (*for our example, this is* 8).

What is the sum of the three circled numbers?

(b) Repeat the steps in part (a) two more times, starting with a different number each time. What do you notice about the sum of the three circled numbers?

(c) Will your result from (b) be true if we create a 3×3 array using different initial numbers in the grey row and column? Explain why it will be true or give an example where it would not be true.

Theme: Number Sense

Problem of the Week

Problem B and Solution

Does This All Add Up?

Problem

In the table shown, the top row and leftmost column are grey, and the remaining numbers form a 3×3 array. Each number in the array is equal to the sum of the numbers in grey in its row and column.

	5	2	7
3	8	5	10
4	9	6	11
6	11	8	13

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5. Circle the remaining number in the array (*for our example, this is* 8).

What is the sum of the three circled numbers?

(b) Repeat the steps in part (a) two more times, starting with a different number each time. What do you notice about the sum of the three circled numbers?

(c) Will your result from (b) be true if we create a 3×3 array using different initial numbers in the grey row and column? Explain why it will be true or give an example where it would not be true.

Solution

(a) The sum of the three circled numbers in our example is $6 + 13 + 8 = 27$.

(b) The sum of the three circled numbers is always 27. Notice that this is also the sum of the six numbers in the grey row and column:
$5 + 2 + 7 + 3 + 4 + 6 = 27$.

(c) Yes, the result will always be true. Regardless of which number is circled first, after the steps are completed there will be exactly one circled number in each row and column. This is because when a number is circled, the other numbers in its row and column are crossed out. Since each number in the array is the sum of the grey numbers in its row and column, it follows that the sum of the three circled numbers will always be equal to the sum of the six grey numbers.

Extension: If each number in the array was instead the *product* of the grey numbers in its row and column, would the product of the three circled numbers equal the product of the six grey numbers?

Problem of the Week

Problem B

Create a Magic Square

In a *magic square*, the sum of the numbers in each row, column, and diagonal is the same. This number is called the *magic sum*. In the following magic square, A, B, C, D, E, F, G, H, and J represent numbers, and clues are given to determine some of these numbers.

- The value of A is one more than the product of 3 and 4.
- The value of B is the sum of 1, 2, and 3.
- The value of C is the smallest odd number greater than 10.
- The value of F is also known as a dozen.
- The value of G is equal to the total number of sides in three triangles.

A	B	C
D	E	F
G	H	J

(a) Use the given clues to determine the values for A, B, C, F, and G.

(b) Determine the magic sum for this magic square, then use this to complete the magic square.

(c) Would you have been able to complete this magic square if only four clues were given, instead of five? If so, which clue(s) do you not need?

Theme: Number Sense

Problem of the Week

Problem B and Solution

Create a Magic Square

Problem

In a *magic square*, the sum of the numbers in each row, column, and diagonal is the same. This number is called the *magic sum*. In the following magic square, A, B, C, D, E, F, G, H, and J represent numbers, and clues are given to determine some of these numbers.

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(a) Use the given clues to determine the values for A, B, C, F, and G.

(b) Determine the magic sum for this magic square, then use this to complete the magic square.

(c) Would you have been able to complete this magic square if only four clues were given, instead of five? If so, which clue(s) do you not need?

Solution

(a) The product of 3 and 4 is $3 \times 4 = 12$. One more than that is $12 + 1 = 13$. Therefore, $A = 13$.

The sum of 1, 2, and 3 is $1 + 2 + 3 = 6$. Therefore, $B = 6$.

The smallest odd number greater than 10 is 11. Therefore, $C = 11$.

A dozen is 12. Therefore, $F = 12$.

Since one triangle has 3 sides, then the total number of sides in three triangles is $3 \times 3 = 9$. Therefore, $G = 9$.

(b) The magic sum is equal to $A + B + C = 13 + 6 + 11 = 30$. Then, since we know A and G, it follows that $D = 30 - A - G = 30 - 13 - 9 = 8$. Since we know C and G, then $E = 30 - C - G = 30 - 11 - 9 = 10$. Since we know B and E, then $H = 30 - B - E = 30 - 6 - 10 = 14$. Finally, since we know C and F, then $J = 30 - C - F = 30 - 11 - 12 = 7$. The completed magic square is shown.

13	6	11
8	10	12
9	14	7

(c) Of the five given clues, the first three clues about A, B, and C are needed to determine the magic sum. Since we need the magic sum to complete the magic square, it follows that we need these three clues.

In our solution for (b), before the final step, we could have determined F using D and E. So it's possible to complete the magic square given only the clues about A, B, C, and G.

Alternatively, we could have first used C and F to determine J. Then we could have used A and J to determine E, and then used E and F to determine D. Then we could have used A and D to determine G, and then used B and E to determine H. So it's also possible to complete the magic square given only the clues about A, B, C, and F. Therefore, we could have completed the magic square without the clue for F or without the clue for G.

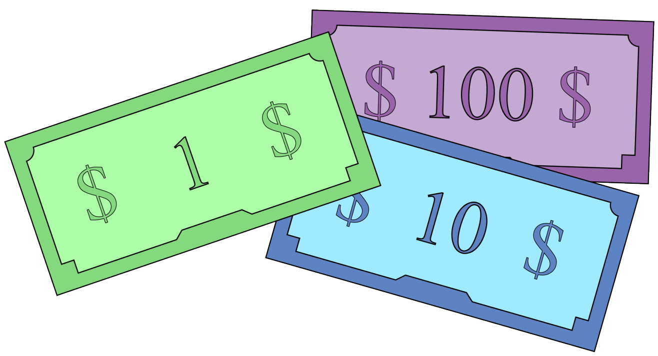
Problem of the Week
Problem B
Ask the Banker

Vesna is the banker in a board game that uses \$1, \$10, and \$100 bills.

(a) Vesna needs to give a player \$2163. How can you do this using the fewest total number of bills? How can you do this using the greatest total number of bills?

(b) Is it possible to give a player \$254 using exactly 20 bills in total? How about using exactly 30 bills in total? If so, show how it's possible. If not, explain why it's not possible.

EXTENSION: Vesna likes when she can give a player the same number of each type of bill. For which total amounts of money is this possible? Explain.



THEMES: ALGEBRA, NUMBER SENSE

Problem of the Week

Problem B and Solution

Ask the Banker

Problem

Vesna is the banker in a board game that uses \$1, \$10, and \$100 bills.

(a) Vesna needs to give a player \$2163. How can you do this using the fewest total number of bills? How can you do this using the greatest total number of bills?

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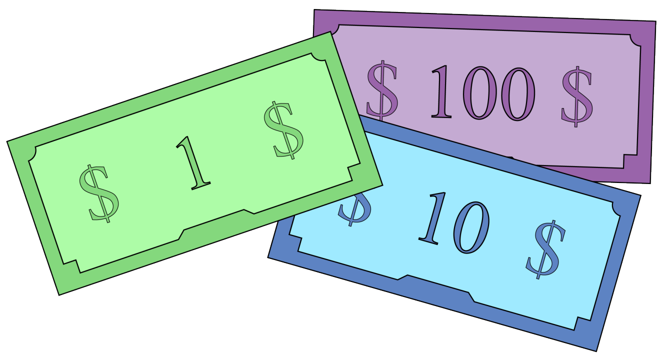
Extension: Vesna likes when she can give a player the same number of each type of bill. For which total amounts of money is this possible? Explain.



Solution

(a) To give a player \$2163 using the fewest total number of bills, we first use as many \$100 bills as we can. Since the number 2163 has 21 hundreds, then we can use at most 21 of the \$100 bills. This gives $21 \times \$100 = \$2100$, so we are left with $\$2163 - \$2100 = \$63$. Next we use as many \$10 bills as we can. Since the number 63 has 6 tens, then we can use at most 6 of the \$10 bills. This gives $6 \times \$10 = \$60$. We are then left with \$3, so we need 3 of the \$1 bills. Thus in total, we use:

$$21 \times \$100 \text{ bills}; \quad 6 \times \$10 \text{ bills}; \quad 3 \times \$1 \text{ bills}$$

This is a total of $21 + 6 + 3 = 30$ bills.

To give a player \$2163 using the greatest total number of bills, we want to use as many \$1 bills as possible. If we use all \$1 bills, then we will use 2163 bills in total.

(b) Using the strategy from (a) to use the fewest total number of bills, we can give \$254 as follows:

$$2 \times \$100 \text{ bills}; \quad 5 \times \$10 \text{ bills}; \quad 4 \times \$1 \text{ bills}$$

This uses a total of $2 + 5 + 4 = 11$ bills. Let's try replacing one \$100 bill with ten \$10 bills. This gives:

$$1 \times \$100 \text{ bill}; \quad 15 \times \$10 \text{ bills}; \quad 4 \times \$1 \text{ bills}$$

This uses a total of $1 + 15 + 4 = 20$ bills, so it is possible to give a player \$254 using exactly 20 bills in total.

Notice that every time we replace a bill with ten smaller bills, the total number of bills increases by 9. This is true if we replace one \$100 bill with ten \$10 bills or if we replace one \$10 bill with ten \$1 bills. So the total number of bills is a sequence that starts at 11 and increases by 9 each time, until it reaches 254 (which is the greatest total number of bills that can be used for \$254). Writing out more terms in the sequence gives 11, 20, 29, $38, \ldots$. Since 30 is not in this sequence, we can *not* give a player \$254 using exactly 30 bills in total.

Solution to Extension: If a player gets 1 of each bill, then the total amount is $1 \times \$100 + 1 \times \$10 + 1 \times \$1 = \$111$. If a player gets 2 of each bill, then the total amount will be $2 \times \$111 = \$222$, because they have twice as many of each bill, so the total amount will double. Similarly, if a player gets 6 of each bill, then the total amount will be $6 \times \$111 = \$666$, and if a player gets 15 of each bill, then the total amount will be $15 \times \$111 = \$1665$. Thus, the total amount will always be a multiple of 111.

We can also use variables to explain this. Suppose a player receives n bills of each type. Then the total amount is equal to $(n \times 100 + n \times 10 + n \times 1)$. This is the same as $n \times (100 + 10 + 1)$, which equals $n \times 111$. Therefore, the total amount must be a multiple of 111.

Problem of the Week

Problem B

Games People Play

Clive and Jill are putting their money together to buy a new GameStation VII. Each of them borrows \$200 from their parents to help cover the cost. In order to encourage their children to repay the money quickly, each of their parents are charging them an extra fee on the last day of each month when there is still money owed. Clive's parents are charging him \$10 extra each month, and Jill's parents are charging her \$12 extra each month.

(a) Clive has decided to pay his parents \$50 on the first day of each month until his loan is repaid, and Jill is paying her parents \$60 on the first day of each month until her loan is repaid. Complete the given tables to determine how long they take to repay their loans.

For Clive:

Month	Payment (\$)	Money Owed (\$)	Extra Fee (\$)
1	50	$200 - 50 = 150$	10
2	50	$150 + 10 - 50 = 110$	10
3			
4			
5			

For Jill:

Month	Payment (\$)	Money Owed (\$)	Extra Fee (\$)
1	60	$200 - 60 = 140$	12
2	60	$140 + 12 - 60 = 92$	12
3			
4			
5			

(b) Who paid their parents more in total?

(c) Is there a monthly payment for Clive which would pay off his loan at the same time as Jill?

Theme: Number Sense

Problem of the Week

Problem B and Solution

Games People Play

Problem

Clive and Jill are putting their money together to buy a new GameStation VII. Each of them borrows \$200 from their parents to help cover the cost. In order to encourage their children to repay the money quickly, each of their parents are charging them an extra fee on the last day of each month when there is still money owed. Clive's parents are charging him \$10 extra each month, and Jill's parents are charging her \$12 extra each month.

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4			
5			

For Jill:

Month	Payment (\$)	Money Owed (\$)	Extra Fee (\$)
1	60	$200 - 60 = 140$	12
2	60	$140 + 12 - 60 = 92$	12
3			
4			
5			

(b) Who paid their parents more in total?

(c) Is there a monthly payment for Clive which would pay off his loan at the same time as Jill?

Solution

(a) The completed tables are shown. Clive paid off his loan in Month 5, while Jill paid off her loan in Month 4.

For Clive:

Month	Payment ($)	Money Owed ($)	Extra Fee ($)
1	50	$200 - 50 = 150$	10
2	50	$150 + 10 - 50 = 110$	10
3	50	$110 + 10 - 50 = 70$	10
4	50	$70 + 10 - 50 = 30$	10
5	40	$30 + 10 - 40 = 0$	0

For Jill:

Month	Payment ($)	Money Owed ($)	Extra Fee ($)
1	60	$200 - 60 = 140$	12
2	60	$140 + 12 - 60 = 92$	12
3	60	$92 + 12 - 60 = 44$	12
4	56	$44 + 12 - 56 = 0$	0

(b) Both Clive and Jill repaid the \$200. In addition, Clive paid $4 \times \$10 = \$40$ in extra fees, while Jill paid only $3 \times \$12 = \$36$ in extra fees. So Clive paid his parents more in total.

(c) At the end of Month 4, Clive still owed \$30, before adding the extra fee. So if Clive could pay a total of \$30 more in the first 4 months, then he could repay the loan in Month 4, like Jill. Since $\$30 \div 4 = \$7.50$, if Clive paid $\$50 + \$7.50 = \$57.50$ each month, then he could repay the loan in Month 4. The table below shows how this would look.

Month	Payment ($)	Money Owed ($)	Extra Fee ($)
1	57.50	$200 - 57.50 = 142.50$	10
2	57.50	$142.50 + 10 - 57.50 = 95$	10
3	57.50	$95 + 10 - 57.50 = 47.50$	10
4	57.50	$47.50 + 10 - 57.50 = 0$	0

Note that \$57.50 is the smallest monthly payment that allows Clive to pay off the loan in Month 4. However, other monthly payments are also possible.

To determine the largest monthly payment that has Clive pay off the loan in Month 4, we would want to have almost all of the loan paid off at the end of Month 3, and so we'd want \$0.01 left after the first 3 payments. Thus, at this point Clive would have paid $\$200.00 - \$0.01 = \$199.99$ of the balance, plus the extra fees from Months 1 and 2, for a total of $\$199.99 + \$20.00 = \$219.99$. Since Clive paid a total of \$219.99 in 3 months, then he paid $\$219.99 \div 3 = \$73.33$ each month. The table below shows how this would work.

Month	Payment ($)	Money Owed ($)	Extra Fee ($)
1	73.33	$200 - 73.33 = 126.67$	10
2	73.33	$126.67 + 10 - 73.33 = 63.34$	10
3	73.33	$63.34 + 10 - 73.33 = 0.01$	10
4	10.01	$0.01 + 10 - 10.01 = 0$	0

Problem of the Week
Problem B
They All Add Up

Etta is finding the sum of the digits of numbers. For example, the sum of the digits in 904 is $9 + 0 + 4 = 13$.

(a) Etta determines that there are 15 integers from 1 to 1000 whose digits have a sum of 4. Find all these integers.

(b) What fraction of these integers are even?

Theme: Number Sense

Problem of the Week

Problem B and Solution

They All Add Up

Problem

Etta is finding the sum of the digits of numbers. For example, the sum of the digits in 904 is $9 + 0 + 4 = 13$.

(a) Etta determines that there are 15 integers from 1 to 1000 whose digits have a sum of 4. Find all these integers.

(b) What fraction of these integers are even?

Solution

(a) First we look at the integers less than 10. The only integer less than 10 whose digits sum to 4 is the number 4 itself.

Next we look at integers between 10 and 99. If the two digits in the integer add to 4, then the digits could be 0 and 4, 1 and 3, or 2 and 2. These pairs of digits and the possible integers they create are summarized in the following table.

Pairs of Digits	Possible Integers
0, 4	40
1, 3	13, 31
2, 2	22

Finally we look at the integers between 100 and 999. Since the digits in 1000 don't have a sum of 4, we can consider only the three-digit numbers. The groups of digits that add to 4 and the possible integers they create are summarized in the following table.

Groups of Digits	Possible Integers
0, 0, 4	400
0, 1, 3	103, 130, 301, 310
0, 2, 2	220, 202
1, 1, 2	112, 121, 211

Therefore the 15 integers whose digits have a sum of 4 are:

$$4,\ 13,\ 22,\ 31,\ 40,\ 103,\ 112,\ 121,\ 130,\ 202,\ 211,\ 220,\ 301,\ 310,\ 400$$

(b) Of these 15 integers, 9 are even. So the fraction of the integers that are even is $\frac{9}{15}$, or $\frac{3}{5}$.

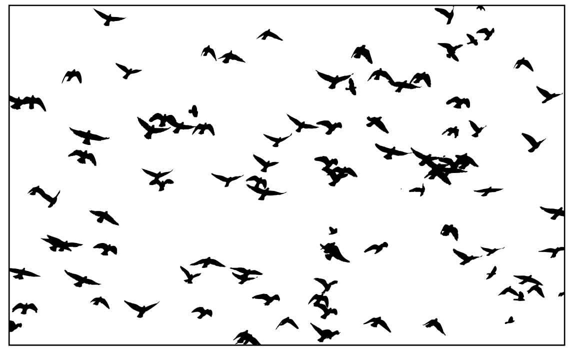
Problem of the Week

Problem B

Destination: Estimation

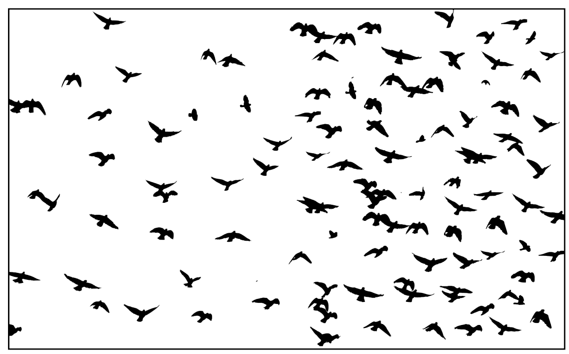
(a) Charley wants to estimate the total number of birds in a picture. Her strategy is to divide the picture into eight smaller rectangles of equal area, count the number of birds in one of the smaller rectangles, and then multiply this number by 8. Using this strategy, estimate the number of birds in Picture 1.

Picture 1



(b) Why might Charley's strategy not give a reliable estimate for the number of birds in Picture 2? Determine a strategy that will give a more reliable estimate, then use your strategy to estimate the number of birds in the picture.

Picture 2



(c) Determine a strategy to estimate the total number of shipping containers on the boat in the picture, then use your strategy to get an estimate. How reliable do you think your estimate is? Explain.

Themes: Geometry & Measurement, Number Sense

Problem of the Week

Problem B and Solution

Destination: Estimation

Problem

(a) Charley wants to estimate the total number of birds in a picture. Her strategy is to divide the picture into eight smaller rectangles of equal area, count the number of birds in one of the smaller rectangles, and then multiply this number by 8. Using this strategy, estimate the number of birds in Picture 1.

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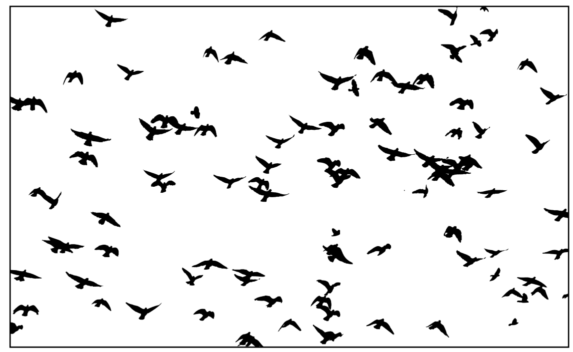
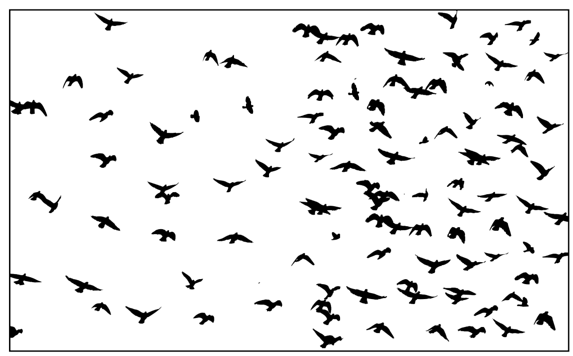


(b) Why might Charley's strategy not give a reliable estimate for the number of birds in Picture 2? Determine a strategy that will give a more reliable estimate, then use your strategy to estimate the number of birds in the picture.

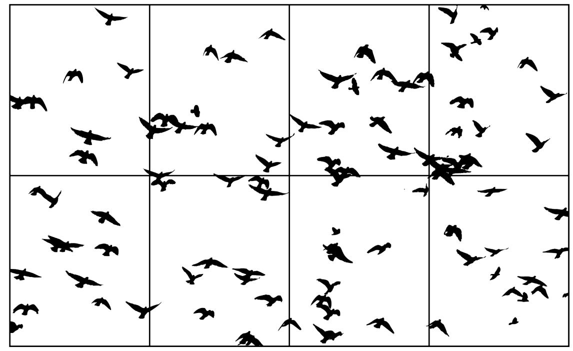
Picture 2



(c) Determine a strategy to estimate the total number of shipping containers on the boat in the picture, then use your strategy to get an estimate. How reliable do you think your estimate is? Explain.

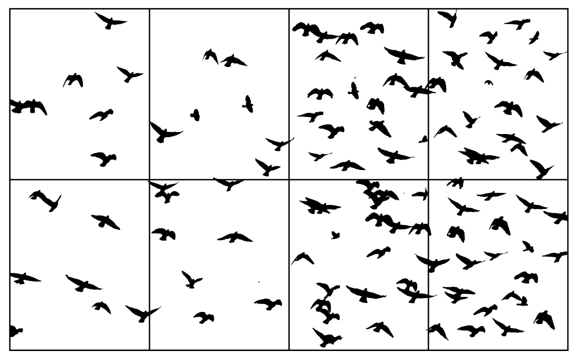
Solution

(a) One way to divide the picture into eight smaller rectangles of equal area is by dividing it into two rows of equal height and four columns of equal width, as shown. We will choose the bottom-left rectangle and count the number of birds in it. Since there are 12 birds, our estimate for the total number of birds in the picture is $12 \times 8 = 96$. Note that estimates will vary depending on which rectangle is chosen and also on how the picture was divided into eight rectangles.



(b) In Picture 2, there are more birds on the right side than the left side. So Charley's strategy could give us an estimate that is considerably larger or smaller than the actual number of birds, depending on which rectangle is chosen.

One strategy is to divide the picture into 8 smaller rectangles like in part (a). The 4 rectangles on the left have few birds, while the 4 rectangles on the right have a lot of birds. We then choose two rectangles; one with few birds and one with a lot of birds, and count the number of birds in each. We then calculate the average (mean) of these and multiply it by the number of rectangles to get an estimate.



In our case, we will choose the top-left and bottom-right rectangles. We counted 7 birds in the top-left rectangle and 22 birds in the bottom-right rectangle. The sum of these is $7 + 22 = 29$ so the average is $29 \div 2 = 14.5$. Then our estimate is $14.5 \times 8 = 116$. Note that estimates will vary depending on the strategy used.

This is more reliable than Charley's strategy because it takes into account the fact that some rectangles have significantly more birds than others.

(c) It is very difficult to determine a reliable estimate for the number of shipping containers because we can't see them all. One strategy is to separate the shipping containers into rows, where each row is parallel to the back of the boat. The only row that we can see clearly is the closest row, where we counted 53 shipping containers. If we assume that the average number of shipping containers in all rows is 53, then we can multiply this by the number of rows to get an estimate. We counted 16 rows, so that means our estimate is $53 \times 16 = 848$. Answers will vary depending on the strategy used, but no strategy can give a very reliable estimate because we can't see so many of the shipping containers.