

Problem of the Week

Problem A and Solution

Puzzling Predicament

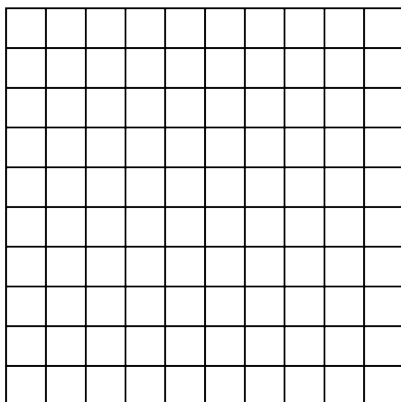
Problem

Pakiza gets a 100 piece puzzle as a gift. When the puzzle is finished, the pieces form a grid, where each row and each column of the grid has the same number of pieces.

- (a) How many pieces are in each row and each column of the puzzle?
- (b) On Monday, Pakiza gathers all the edge and corner pieces and puts them together to form the outside edge of the puzzle. How many edge and corner pieces does the puzzle have in total?
- (c) Pakiza continues to work on the puzzle a little bit every day. She places 10 pieces in the puzzle on Tuesday. On each day after that, she places one more puzzle piece than the day before, until the puzzle is complete. So, she places 11 pieces on Wednesday, 12 pieces on Thursday, and so on until all the pieces are in place. On what day of the week does Pakiza complete the puzzle? Justify your answer.

Solution

- (a) We know that each row and each column of the puzzle have the same number of pieces, and there are 100 pieces in total. Since $10 \times 10 = 100$, then each row and each column of the puzzle must contain 10 pieces.
- (b) It might help to look at a 10 by 10 grid in order to determine the total number of edge and corner pieces.



There are 4 corner pieces. Not counting the corners, there are 8 edge pieces on each side of the puzzle. So the total number of edge and corner pieces is: $4 + 8 + 8 + 8 + 8 = 36$.



Note: This is a classic example of the *fencepost problem*. If we are not careful, we might think that a 10×10 grid has $4 \times 10 = 40$ squares around its edge. However, this counts each corner piece twice, since a corner piece is part of two sides of the puzzle. The fencepost problem describes a situation where you may be off-by-one when counting. It can cause problems when writing computer programs that include some repetition.

- (c) We can make a table to keep track of how many pieces are filled in the puzzle over time. The first entry in the table will be on Monday when Pakiza assembles the 36 pieces to form the outside edge of the puzzle.

Day	Number of Pieces Added	Total Pieces
Monday	36	36
Tuesday	10	46
Wednesday	11	57
Thursday	12	69
Friday	13	82
Saturday	14	96

At this point, there are only 4 pieces left in the puzzle. So, we know Pakiza will complete the puzzle on the next day, which is Sunday.