



## Problem of the Week

### Problem A and Solution

### Team Tournament

#### Problem

Ms. McNeil organizes a bean bag tic-tac-toe tournament for her class. She divides her class into 6 teams. When one team plays another team during the tournament, this is called a match.

- (a) In the tournament, each team has to play every other team once. How many matches will each team play during the tournament? Justify your answer.
- (b) How many matches are played in total during the tournament? Justify your answer.

#### Solution

- (a) Since each team has to play every other team once during the tournament, each team must play the other 5 teams. In other words, each team has 5 matches.
- (b) We will call the teams Team 1, Team 2, Team 3, Team 4, Team 5, and Team 6. One way to solve this problem is to write out all the matches, being careful to make sure we do not include duplicate matches. For example, when Team 1 plays Team 2, that match is the same as saying Team 2 plays Team 1.

The possible matches are:

- Team 1 and Team 2, Team 1 and Team 3, Team 1 and Team 4, Team 1 and Team 5, Team 1 and Team 6
- Team 2 and Team 3, Team 2 and Team 4, Team 2 and Team 5, Team 2 and Team 6
- Team 3 and Team 4, Team 3 and Team 5, Team 3 and Team 6
- Team 4 and Team 5, Team 4 and Team 6
- Team 5 and Team 6

Therefore, there are 15 matches in total in the tournament.

Alternatively, since each team has to play 5 matches and there are 6 teams, we can count the total number of matches in the tournament as  $5 + 5 + 5 + 5 + 5 + 5 = 5 \times 6 = 30$ .

However, this calculation actually double counts matches. For example, it counts the match between Team 1 and Team 2 as two separate matches (one match from the point of view of Team 1 and one match from the point of view of Team 2). This is true for every match between two teams. So, to get the true count of the total number of matches in the tournament, we need to divide by 2 to get  $30 \div 2 = 15$  matches.