



Problem of the Week

Problem A and Solution

Colouring Box

Problem

Mysha was playing with a blank 10 by 10 grid on white paper.
She wants to create a pattern that follows these rules:

The top left square of the grid is square 1.

Mysha counts along a row from left to right. From the end of one row, she moves to the leftmost square in the row below and continues counting.

Starting with the 5th square on the top row, she adds red to every 5th square.

Starting with the 7th square on the top row, she adds blue to every 7th square.

Starting with the 9th square on the top row, she adds yellow every 9th square.

If a square is part of more than one pattern, then the colours will mix together to make a new colour, according to the following rules:

- Red mixed with blue will make purple.

$$\boxed{\text{R}} + \boxed{\text{B}} = \boxed{\text{P}}$$

- Blue mixed with yellow will make green.

$$\boxed{\text{B}} + \boxed{\text{Y}} = \boxed{\text{G}}$$

- Yellow mixed with red will make orange.

$$\boxed{\text{Y}} + \boxed{\text{R}} = \boxed{\text{O}}$$

- (a) How many squares of each colour will there be in the grid when Mysha finishes filling in the patterns?
- (b) After finishing her colouring, how many squares of the grid are still white?



Solution

The grid below shows the result of following the pattern rules:

				R		B		Y	R
			B	R			Y		R
B				R		Y	B		R
				P	Y				R
	B			O				B	R
			Y	R	B				R
		G		R					P
	Y			R		B			R
Y			B	R					O
B				R			B	Y	R

An interactive version of the pattern can be found here:

<https://openprocessing.org/sketch/2537459>

- (a) From the completed grid, we count the number of each colour:

16 red, 11 blue, 8 yellow, 2 purple, 1 green, 2 orange

Another way to calculate this is to use skip counting to keep track of the squares that are filled with colour and look for overlapping numbers to see where the colours purple, green, and orange will appear. We cross out the duplicates from the primary colours (red, blue, and yellow) to get the final tally of each colour.

Red squares:

5, 10, 15, 20, 25, 30, ~~35~~, 40, ~~45~~, 50, 55, 60, 65, ~~70~~, 75, 80, 85, ~~90~~, 95, 100

Blue squares: 7, 14, 21, 28, ~~35~~, 42, 49, 56, ~~63~~, ~~70~~, 77, 84, 91, 98

Yellow squares: 9, 18, 27, 36, ~~45~~, 54, ~~63~~, 72, 81, ~~90~~, 99

Purple (red and blue) squares: 35, 70

Green (blue and yellow) squares: 63

Orange (yellow and red) squares: 45, 90



Another way to calculate these totals is to consider common multiples. To start, we can calculate that there are $100 \div 5 = 20$ squares that will be coloured in red, $100 \div 7 = 14$ (remainder 2) squares that will be coloured in blue, and $100 \div 9 = 11$ (remainder 1) squares that will be coloured in yellow. Next we consider the common multiples of 5 and 7 that are less than or equal to 100. Since $5 \times 7 = 35$, then we can skip count by 35 to find these numbers, 35 and 70, for a total of 2 purple squares. Similarly the common multiples of 5 and 9 that are less than 100 are 45 and 90, for a total of 2 orange squares. Finally, the only common multiple of 7 and 9 that is less than 100 is 63, for a total of 1 green square.

Again, we can eliminate the duplicates from the primary colours count. We eliminate 4 from the number of red squares for a total of $20 - 4 = 16$. We eliminate 3 from the number of blue squares for a total of $14 - 3 = 11$. We eliminate 3 from the number of yellow squares for a total of $11 - 3 = 8$. The lowest common multiple of 5, 7, and 9 is $5 \times 7 \times 9 = 315$, which is greater than 100. So none of the squares in the grid will be filled in with all three primary colours.

- (b) Since the grid is 10 by 10, there are a total of $10 \times 10 = 100$ squares. In part (a), we found a total of $16 + 11 + 8 + 2 + 1 + 2 = 40$ squares filled with some colour. This means there is a total of $100 - 40 = 60$ white squares left in the grid.